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Moment-curvature-thrust relationships in hybrid members

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In order to overcome the difficulties encountered in closed form solutions, moment-curvature-thrust relationships are developed for hybrid and nonhybrid cross sections utilizing an open form method. The use of horizontal sectors permits the inclusion of residual stresses and/or nonbilinear stress-strain relationships, if desired. Theoretical and experimental data are compared. Applications to circular tubes and other cross sections are discussed.

Results indicate that open form solutions are feasible for calculating moment-curvature-thrust data. Hybrid cross sections are easily treated by open form solutions.

MOMENT-CURVATURE-THRUST RELATIONSHIPS

IN HYBRID MEMBERS

by

DOUGLAS WRENN FIALA

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TO MY

PARENTS

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and
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NOMENCLATURE

SYMBOL	UNITS	DEFINITION
AR		Residual Stress Constant
ASTEP	in.	Axis Increment
BR		Residual Stress Constant
CR		Residual Stress Constant
CSTR	in./in.	Nonbilinear Stress-Strain Dividing Strain
ECG	in.	Distance to Elastic Centroid
EMIN	K/in^2	Minimum Elastic Modulus
FY	K/in^2	Assumed Yield Stress
h1,h2	in.	Lower and Upper Sector Heights
I		Iteration Increment
J		Iteration Increment
KF		Angle Loop Parameter
KL		Angle Loop Parameter
KS		Angle Loop Parameter
L		Iteration Incrementer
M	K-in.	Bending Moment acting on the Cross Section
M_i	K-in.	Bending Moment influencing the i^{th} Stress Block
M_p	K-in.	Ultimate Bending Moment
M_y	K-in.	Nondimensionalized Bending Moment
N		Number of sections in the Cross Section
NN2		Number of Linear Approximations in one-half of the Nonbilinear Stress-Strain Relationship

SYMBOL	UNITS	DEFINITION
P	K	Axial Thrust Acting on the Cross Section
P_i	K	Axial Thrust Acting on i^{th} Stress Block
P_y	K	Nondimensionalized Axial Thrust
PCG	in.	Distance to the Plastic Centroid
PM	K-in.	Plastic Bending Moment
SF		Angle Loop Parameter
s	K/in^2	Stress
s1,s2	K/in^2	Lower and Upper Stress Intensities
SF1		Shape Factor
TWD	in.	Transformed Section Width
[X]		Iteration Vector
[Y]		Iteration Matrix
y	in.	Distance from Moment Center to Point of Stress
y_i	in.	Distance from Moment Center to Centroid of i^{th} Sector
w1,w2	in.	Lower and Upper Sector Widths
ZCG	in.	Distance to Infinite Zero Thrust Axis
Ø	Rad.	Angle of Curvature
$Ø_k$	Rad.	General Angle of Curvature
$Ø_s$	Rad.	Angle of Curvature whose Zero Thrust Axis Lies on the PCG
ε_y	in./in.	Strain that Produces the Stress, FY

INTRODUCTION

1.1 GENERAL

The advent of plastic analysis of structural members and braced frames has significantly broadened the scope of structural design. Because plastic design deals with ultimate member strength, information is needed for all possible stress conditions. Such information is available for most wide flange cross sections in the form of moment-curvature-thrust curves but is rare for most other structural shapes. Moment-curvature-thrust relationships are hereinafter referred to as M-Ø-P relationships.

Although elastic deflections are determined routinely by small deflection theory, deformations and displacements in the elasto-plastic range may not be determined as easily because they depend on the changing angles of curvature of the cross section over the member length. Deflections are obtained by integrating these angles of curvature. Ultimate capacity is attained when all fibers of the cross section are completely plastified. All axial thrust and radius of curvature combinations require a finite bending moment to cause full plastification. If axial thrust and bending moment variations are known, the changing radii of curvature (strain slopes) are defined, and deflections are obtained as outlined above.

Elasto-plastic response depends on material properties as well as member geometry. Members with similar configurations, such as wide flange shapes, generally exhibit similar M-Ø-P relationships. However, these relationships are usually not available for all different cross sectional shapes. The possibility of finding such pertinent data is further reduced when hybrid members are used. A hybrid member is one constructed of two or more interconnected materials. The most common use of hybrid construction involves two different grades of steel, and is generally encountered in plate girders.

In some instances both strength and member geometry must be considered in design. If the selection of dimensions are left to the structural designer, no major problems are encountered. However, if sizes are limited by architectural considerations, common rolled shapes may not satisfy the design requirements. The engineer may have to resort to different structural shapes, such as tubes, built-up and hybrid members. Furthermore, the problem may be complicated by the presence of materials with nonlinear stress-strain characteristics.

If deflections are to be calculated and interaction curves produced, accurate M-Ø-P relationships are necessary. Generation of M-Ø-P relationships by longhand procedures is tedious even for the simplest shapes. Therefore, there exists a need for quick and accurate means of determining these relationships.

1.2 PROBLEM DEFINITION

Methods of producing M-Ø-P relationships include:

a) Closed Form Solutions

b) Open Form Solutions

a. Closed Form Solutions

Closed form solutions, capable of incorporating idealized linear residual stress patterns were introduced for wide flange sections by Ketter, Kaminsky and Beedle (5)*. Their treatment required the use of an idealized bilinear stress-strain relationship, (Fig. 1a), with identical compressive and tensile stress properties. For closed form solutions, sets of bounded relationships are derived for the angle of curvature, axial thrust and bending moment associated with the cross section. The equation sets are bounded in that each set is valid only for one elasto-plastic stress condition. Figures 2b through 2f depict a wide flange member in progressive elasto-plastic stages. Figure 2b shows the cross section in a fully elastic condition while Fig. 2f illustrates complete plasticity caused by bending. Since the fully plastic stage (Fig. 2f) is a special case of the elasto-plastic condition shown in Fig. 2e, only four sets of relationships are required for a wide flange member. Thus one elastic and three elasto-plastic conditions must be considered in deriving the necessary equations. These equations are then "nondimensionalized" by dividing curvature, thrust and moment equations by predetermined curvature, thrust and moment values. The nondimensionalization eliminates plotting wide ranges of curvature, thrust and moment values when M-ϕ-P diagrams are constructed.

*Numbers listed refer to references at the end of this thesis.

For the purpose of this investigation a section is defined as a rectangular member component. For example, the member in Fig. 2a is composed of three sections. In general, if a member is composed of N sections, a total of 3N+3 equations are needed for curvature, thrust and bending moment calculations. To generate the M-ϕ-P data, the desired stress condition from Figure 2 is first determined and then the corresponding relationships are used to construct auxiliary curves as shown in Fig. 3. The M-ϕ-P diagram (Fig. 4) is then plotted using data from the auxiliary curves.

As previously mentioned, closed form solutions use bilinear stress-strain relationships because definite yield points are necessary for the bounded equations. In previous treatments, linear residual stress idealizations (Fig. 5a) have been adopted from the approximately parabolic residual stress patterns shown in Fig. 5b. This simplification is necessary because the manipulation of parabolic shapes in bounded equations is prohibitively difficult to treat mathematically. Similar arguments have also eliminated the consideration of nonbilinear stress-strain relationships by closed form methods.

It should be recognized that closed form solutions are not limited to wide flange members. Samani (8) presented relationships for general residual stress patterns in his study of M-ϕ-P relationships for built-up steel members. In the same vein, Matthys (6) presented closed form relationships for hollow circular tubes. From their work it is apparent that the more general the treatment, the more tedious the longhand application becomes. It is also possible to derive relationships for other conventional and hybrid cross sections in the same manner.

It is apparent that closed form solutions have many disadvantages as follows:

(1) The correct stress condition must be determined before data is calculated.

(2) Only linearly varying residual stress distributions can be treated.

(3) Nonbilinear stress-strain relationships tend to complicate the derivations, and are not feasible to incorporate.

(4) Closed form solutions for wide flange members are easily solved by digital computers,but as more complicated shapes and residual stress patterns are introduced, even computer solutions may become awkward.

(5) Derivation of equations for more complicated cases could be prohibitively long.

b. Open Form Solutions

Having considered the previously mentioned shortcomings, Erzurumlu, Bakir and Kok (1) initially proposed the concept of open form solutions for calculating M-Ø-P data. An open form solution is a generalized computational procedure capable of treating any forseen combination of factors influencing the cross sectional behavior. Some of these influencing factors are:

(1) The use of materials with varying properties within the cross section;

(2) Residual stress patterns in any or all sections;

(3) Nonbilinear stress-strain relationships for any or all materials;

(4) Any combination of yield stress patterns;

(5) Any combination of the above factors.

In open form solutions one calculation procedure treats all possible stress conditions. The open form approach as presented recently by Fiala and Erzurumlu (3) has proved useful in calculating M-Ø-P data for circular tubes permitting the consideration of both residual stress patterns and nonbilinear stress-strain relationships. Open form solutions are feasible for most cross sectional configurations and seem to be applicable to most hybrid members.

1.3 OBJECTIVES OF INVESTIGATION

The purpose of this investigation is to develop an open form computer solution for calculating M-Ø-P data for cross sections of rectangular configurations. Nonbilinear stress-strain relationships and general residual stress patterns are treated in both homogeneous and hybrid members. Open form solutions are advantageous because one programmed solution package is capable of treating many different types of cross sections. Steps taken to achieve the stated objectives are as follows:

(1) Investigation of bending moment axes and symmetry relationships;

(2) Development of a mathematical model;

(3) Discussion of analytical procedures;

(4) Development of a solution package;

(5) Comparison of results with experimental data.

THEORETICAL CONSIDERATIONS

2.1 GENERAL

The calculation of M-Ø-P relationships requires knowledge of the proper bending moment axis. The position of this axis depends on geometric and material symmetry. Geometric symmetry requires identical dimensions on both sides of the horizontal bending axis (Fig. 6a). Material symmetry is achieved when geometrically symmetric sections are constructed of identical materials (Fig. 6b). In this investigation, it is assumed that the cross section possesses both geometric and material symmetry about a vertical axis.

Locations of plastic and elastic centroids of a cross section are governed by variations of geometry and materials. If material and geometric symmetry exist about a horizontal axis, the two centroids coincide and the cross section is referred to as totally symmetric. Various examples of totally symmetric cross sections are shown in Figure 7a.

2.2 DEFINITION OF VARIOUS CASES

A proper moment center can now be considered for combinations of axial thrust and bending moment. If only elastic stresses influence the totally symmetric cross sections shown in Fig. 7a, the elastic centroid can be used as the proper moment center. On the other hand, when the cross section is totally plastified, the plastic centroid is used. For totally symmetric cross sections the cause of total plasticity is unimportant in that the moment center remains at the plastic centroid whether total plasticity is caused by axial thrust (Fig. 8a), bending moment (Fig. 8b) of a combination of the two (Fig. 8c). The coincident centroid position can be used as the moment axis for elastic, elasto-plastic and plastic stress configurations.

Analysis of the asymmetric cross sections shown in Fig. 7b is more complex because the elastic and plastic centroids are not coincident and their treatment is beyond the scope of this investigation. The elastic centroid is used as the moment center for elastic stress conditions. Therefore, only pure bending cases will be considered for asymmetric cross sections. Stresses produced by pure bending are easily treated since all moment centers yield identical bending moments for a couple. For all angles of curvature a stress block can be constructed such that no axial thrust influences the cross section. The neutral axis of this stress block is defined as the zero thrust axis. For totally symmetric cross sections all zero thrust axes lie at the coincident centroids.

For asymmetric cross sections the positions of the zero thrust axes vary between the elastic centroid (ECG) and the infinite zero thrust axis (ZCG). The ZCG is defined as the position of the zero thrust axis for an assumed infinite angle of curvature. At this stage the cross section is totally plastified by bending moments, and the stress block contains equal tensile and compressive component volumes. For example, the zero thrust axes for the asymmetric cross section shown in Fig. 9 vary between the ECG and ZCG for the angles of curvature indicated by $\emptyset/\emptyset_y$. Further implications concerning the shift of the zero thrust axis are discussed in Appendix A.

The computation of theoretical M-Ø data for asymmetric cross sections requires the location of zero thrust axes for all considered angles of curvature. These axes are easily determined by simple iterative procedures since they always lie between the ECG and ZCG.

2.3 LIMITATIONS

All totally symmetric cross sections are treated for all axial thrust percentages. Only pure bending is considered for asymmetric cross sections. The possibility of further treatments of asymmetric cross sections is discussed in Appendix A.

2.4 MATHEMATICAL MODEL

For numerical calculations, a cross section is divided into a finite number of sectors (Fig. 10a). A section can be composed of more than one sector but a sector can contain area from only one section. Upper and lower sector boundaries are defined as stations. The stress block (Fig. 10b) influencing a cross section is numerically defined by calculating the height, stress intensity and width of the cross section at all stations. The bending moment and axial thrust contributions of all sectors are computed and added to produce the forces influencing the cross section.

A linear variation of stress is assumed between stations for moment and thrust calculations. Bilinear stress-strain relationships and linear residual stresses lie within this assumption but the calculation procedure must be altered for nonbilinear stress-strain relationships and/or nonlinear residual stresses. The number of stations needed to define the stress block volume for linear cases with no residual stress is a function of the number of sections in the cross section. A maximum of three station coordinates, two for sector boundaries and one for a yield point, could be required for each sector. For the general stress condition two additional stations could be required; one for a zero stress point and another for a second yield point in one sector. If the cross section contains N sections then 3N+2 stations could be required to define the stress block. If linear residual stresses are considered, the number of additional stations required depends on the linear variations and no general rule can be given for the required additional stations.

Nonbilinear stress-strain relationships (Fig. 1b) and/or nonlinear residual stresses (Fig. 5b) produce a nonlinearly varying stress distribution as shown in Fig 11a. This distribution violates the linearly varying stress assumption made for thrust and bending moment calculations. The treatment of this nonlinear case is based on approximating the general stress distribution with a series of linear approximations (Fig. 11b) that adequately estimates the actual relationship. An adequate number of sectors are placed in nonlinearly varying sections to produce the linear approximations. It should be noted that the method of producing the stress distribution dictates numerical accuracy. The remaining calculations, however, are independent of the stress block formation procedures.

2.5 GOVERNING EQUATIONS

The determination of thrust and bending moment relationships are based on the following well known equations of statics:

$$P = \int_A sdA \qquad [1]$$

$$M = \int_A ysdA \qquad [2]$$

where P = the thrust acting on the cross section;

M = the bending moment acting on the cross section;

s = the stress at any point on the cross section;

y = the distance from the moment center to the point of stress.

Each sector forms a finite stress block as shown in Fig. 10c. The thrust and moment contribution of each sector block can be defined as:

$$P_i = \int_{h1}^{h2} swdh \qquad [3]$$

$$M_i = \int_{h1}^{h2} yswdh = y_i P_i \qquad [4]$$

where P_i = the thrust acting on the i^{th} stress block;

M_i = the moment caused by the i^{th} stress block;

y_i = the distance from the moment center to the centroid of the i^{th} stress block;

h1, h2 = lower and upper station coordinates, respectively.

Considering the contributions of all sector stress blocks, the axial thrust and bending moment acting on the cross section can be represented as:

$$P = \sum_{i=1}^{n} P_i \qquad [5]$$

$$M = \sum_{i=1}^{n} y_i P_i = \sum_{i=1}^{n} M_i \qquad [6]$$

where n = the number of sectors used to build the stress block volume.

2.6 NONDIMENSIONALIZATION

In an effort to nondimensionalize the M-Ø-P diagrams the following constants are defined. The thrust, P_y, is the thrust acting on the cross section when full plasticity is achieved by axial loading. The angle of curvature, $Ø_y$, is defined as the angle of curvature that causes the first assumed plastic stress at any fiber when the neutral axis passes through the elastic centroid. The moment, M_y, is the bending moment influencing the member when the strain angle $Ø_y$ is used. As a convenient check for output data, the nondimensionalized moment, M/M_y, should equal unity when $Ø=Ø_y$.

ANALYTICAL PROCEDURE

3.1 GENERAL

M-ϕ-P diagrams show moment-curvature relationships for constant percentages of the thrust, P_y. A moment is computed for each selected radius of curvature and each selected thrust percentage. For each strain condition associated with a radius of curvature, a corresponding stress diagram is constructed such that a certain thrust percentage acts on the cross section. The axial thrust and the moment acting on the cross section can then be calculated using Eqs. 5 and 6 from which nondimensionalized values of P/P_y, M/M_y and ϕ/ϕ_y can be obtained.

The problem here is to determine the location of the neutral strain axis that yields the selected thrust percentage. Suppose the neutral axis originally passes through point 0 in Fig. 12a. If the calculated thrust percentage is not equal to that anticipated, the neutral axis is shifted either up or down along the vertical member axis and the thrust is recalculated. The proper neutral axis position is approached by a succession of axis shifts indicated by the points 0,1,2,...,n. When the proper neutral axis position is obtained, the bending moment can be calculated. The accuracy of the thrust estimation controls the reliability of the calculated bending moment for the selected thrust values. This process has proven to be computationally inefficient, especially for high thrust percentages associated with small angles of curvature where the neutral axis lies off the cross sectional area.

Another approach is to select the initial position of the neutral axis at the zero thrust axis (Fig. 12c). After the thrust (initially equal to zero) and the bending moment are calculated, the neutral axis is raised by a predetermined amount, ASTEP, and new thrust and moment values are determined and stored for future reference. The shift of the neutral axis is terminated when the calculated thrust reaches within two percent of P_y. The values stored for reference are nondimensionalized thrust and moment percentages, and if plotted, result in thrust-moment relationships for constant angles of curvature, ϕ/ϕ_y, as shown in Fig. 13. If graphs similar to those shown in Fig. 13 are drawn, the moment corresponding to any thrust percentage can be graphically estimated with suitable accuracy. However, instead of constructing graphs, the open form solution employs iterative procedures to estimate the desired moments. Since the neutral axis is raised by predetermined increments, neither thrust nor moment values will be equally spaced tabulated points, and therefore, equal point interpolation formulas cannot be used. Aitken's process (7), a nonequally spaced linear interpolative procedure, is used for the iteration. It should be noted that the range around the desired point should be chosen for the iteration, thus increasing numerical accuracy. Iterations utilizing a complete graphical range lead to erroneous results because the thrust-moment relationship cannot be well represented by a single function. Series of short range approximations as shown in Fig. 14 are more appropriate. When a range is chosen around the desired moment value, Aitken's process adapts a function to that protion of the curve.

3.2 THE COMPUTER SOLUTION

The computer program for treating totally symmetric cross sections is listed in Appendix C.1 and is flow charted in Appendix D.1. The program for asymmetric cross sections is also listed in Appendix C.2 and flow charted in Appendix D.2. In both cases any number of cross sections may be treated in one run. Geometric and material inputs are identical for both processes and preliminary control calculations are similar.

Both procedures calculate preliminary values of thrust (P_y), moment (M_y), angle of curvature ($\emptyset_y$), distance to the elastic centroid (ECG), plastic moment (PM) and the shape factor (SF1). For asymmetric cross sections the additional parameters EMIN (minimum elastic modulus), TWD (transformed section widths), ZCG (distance to the infinite zero thrust axis) and PCG (distance to the plastic centroid) are determined.

After data has been input and preliminary calculations made, a series of loops is commenced for various strain diagram slopes defined by the inputs KF (initial loop value), KL (final loop value), KS (loop increment) and SF (a dividing constant which, if equal to four, divides the loop into increments of one-fourth $\emptyset_y$). The neutral axis is incremented by intervals of ASTEP as shown in Fig. 12c and moments corresponding to predetermined thrust percentages are calculated. Geometric and sectional properties, nonbilinear stress-strain relationships, residual stress constants and M-Ø-P data are printed in tabular form as shown in Fig. 15.

The asymmetric procedure varies from the totally symmetric process in calculating more preliminary constants. Asymmetric cross sections also require the determination of the zero thrust axes positions for various angles of curvature.

As revealed by the flow charts, both programs employ subroutines and driver subsections for most calculation procedures. The subroutines are listed in Appendix C.3 with driver subsections in Appendix C.4. Driver subsections are used mainly for preliminary constant calculations and for sections common to both totally symmetric and asymmetric programs. Flow diagrams are given for subroutines and driver subprograms in Appendices D.3 and D.4, respectively. Brief explanations of the subroutines follow.

3.3 SUBROUTINES

Subroutine RSTRS treats quadratic, linear and constant residual stress patterns. The constants AR, BR and CR are input for all sections and correspond to A, B, and C, respectively, in the quadratic equation $AX^2+BX+C=0$. The constants are input to define the residual stress patterns for a section as a function of the station heights over the section. Appropriate constants are input as zero for nonparabolic stress idealizations. Because vertical sectors are not used, stress variations are not allowed over the width of the flange. Variations over the depth of the flange can be treated, however.

Subroutine XSTRS calculates stresses from given strains for materials having nonbilinear (Fig. 1b) stress-strain relationships. A series of linear approximations are programmed in lieu of exact stress-strain functions. Figure 16 graphically represents such approximations which are governed by NN2 defined linear portions in half the stress-strain diagram. The length of each approximation is governed by the strain range, CSTR. Equal numbers of compressive and tensile approximations are used. If strains outside the defined range are encountered, a signed constant stress, FY, is assigned. Although this procedure is valid for steel and some other materials, it is unacceptable for materials exhibiting stress-strain relationships as shown in Fig. 17. In this instance, yield stresses are input twice for all materials. Initial values are used for preliminary constant calculations. If stress-strain relationships similar to that of Fig. 17 are used, second yield stresses are input as zero to accommodate material fracture.

Subroutine SDIAG numerically defines the stress block volume in the computer. If only linear functions are used in volume construction, upper and lower station data are calculated for each section. If nonlinearities are employed, data are calculated for the number of sectors specified for each section by the programmer. Subroutines RSTRS and XSTRS are used in defining nonlinear cases. If necessary, yield point stations are provided for all sections. If nonlinear calculations govern a section, sufficient linearity is assumed and the yield point search is abandoned. Data for zero stress points are calculated as needed for linear sections.

Subroutine MOMNT calculates the bending moment contribution of each sector stress block and forms the resultant moment. Counterclockwise moments are considered positive. Subroutine TSTRS sums block volumes to obtain the axial thrust influencing the cross section. Positive stresses are treated as tension.

Subroutine SHIFT, used in conjunction with subroutine SDIAG, stores station data correctly in the dimensioned files. Subroutine AITKN is used to determine bending moments and zero thrust axes from calculated data points. A more complete explanation of Aitken's process and subroutine AITKN are presented in Appendix B.

NUMERICAL EXAMPLES AND TEST COMPARISONS

4.1 WIDE FLANGE MEMBER

Open form M-Ø-P relationships are compared to the well known closed form results for a W 8X31 published by Ketter, Kaminsky and Beedle (5). Such a comparison verifies open form theoretical accuracy. Cross sectional data and computer printout are given in Fig. 18. Both open and closed form solution M-Ø-P relationships are identical and are given in Fig. 19. Bilinear stress-strain relationships were used for both solutions.

4.2 HYBRID MEMBERS

a) GENERAL

General experimental data for M-Ø-P relationships for hybrid cross sections are not available for comparison. Erzurumlu (2) has conducted experiments dealing with the moment-curvature relationships for selected hybrid cross sections. Theoretical open form results are compared to Erzurumlu's experimental data.

The static yield points as determined by cupon tests (2) are 40 KSI, 57.5 KSI and 100 KSI for A-36, A-441 and SSS-100 steels, respectively. Since no strain hardening is considered the theoretical ultimate moments of the open form solution should be somewhat less than the measured ultimate moments.

b) DATA COMPARISON

Cross section T1 is composed of an A-36 web and A-441 flanges. Experimental and theoretical M-Ø relationships are shown in Fig. 20. For no thrust, the calculated plastic moment is M_p = 426 K-IN. The two curves are nearly identical in the elastic range but vary by up to 20% in the plastic region where the ultimate measured moment is 501 K-IN. The open form solution is conservative for bilinear stress-strain relationships. Actual stress-strain relationships produce more accurate results. M-Ø-P data is printed in Fig. 21 and the M-Ø-P diagram is shown in Fig. 22.

Cross section T2 has an A-441 web and SSS-100 flanges. Experimental and theoretical M-Ø curves are shown in Fig. 23. The expected plastic moment was M_p = 1054 K-IN. and failure occurred at 1290 K-IN., or a 23% increase. Again, bilinear stress-strain relationships were used in data calculations. The M-Ø-P diagram is given in Fig. 24.

Cross section T3 is asymmetric with an A-441 web and top and bottom flanges of SSS-100 and A-36 steel, respectively. Experimental and theoretical M-Ø curves are given in Fig. 25. The expected plastic moment was M_p = 595 K-IN. but the cross section withstood 987 K-IN. Bilinear stress-strain relationships were used in data calculations.

4.3 EXTENSION TO CIRCULAR TUBES

Since circular tubes are totally symmetric (Fig. 7a), a computer program similar to the totally symmetric program is used. Variations occur in preliminary constant calculations and in subroutines SDIAG, MOMNT and THRST. Nonparallel sector walls (Fig. 26a) necessitate changes in the subroutines.

When lines ac and bd are considered to be straight, a rhombus is formed as shown in Fig. 26a. The resulting stress block volume is shown in Fig. 26b. Because w1 and w2 are not equal, changes must be made in the moment and thrust calculation procedures.

Subroutine SDIAG is altered to account for changing circular widths. While ovaling (4) has not been considered, it could easily be incorporated in the subroutine. Because of ovaling, computed data for large angles of curvature, ϕ/ϕ_y, may, in most cases, overestimate actual behavior.

The circular tube program is listed in Appendix C.5 and is flow charted in Appendix D.5. M-ϕ-P data for a 14 in. O.D., .5 in. wall thickness standard tube is given in Fig. 27 and the results are plotted in Fig. 28.

4.4 OTHER CROSS SECTIONS

As long as cross sectional widths are definable in terms of station coordinates no undue hardship is forseen in treating any cross section by open form solutions. The two possible configurations for stress block volumes are solved in the totally symmetric program (parallel vertical walls) and the tube program (nonparallel vertical walls). Subroutines MOMNT and THRST will never vary from these two cases. Subroutine SDIAG will need modification if cross sections other than vertical parallel walled cross sections or tubes are encountered.

CONCLUSIONS AND RECOMMENDATIONS

5.1 CONCLUSIONS

Due to the success and applicability of open form solution techniques, the following conclusions may be drawn.

1. Open form computer solutions can be successfully used for tubes, hybrid and nonhybrid cross sections, and can be extended to almost any cross sectional configuration.
2. General stress-strain relationships can be handled by the proposed mathematical model.
3. Nonlinear residual stress patterns can also be incorporated.
4. The use of bilinear stress-strain relationships is conservative and more accurate results could be obtained by using general stress-strain relationships.
5. The theoretical results generated by this solution are generally limited to low values of ϕ/ϕ_y as at higher values of ϕ/ϕ_y local buckling and ovaling may occur.

5.2 RECOMMENDATIONS

1. Methods of theoretically uncoupling bending moments and axial thrust from a plastic stress distribution need to be developed, thus leading to the proper moment center for elasto-plastic stress conditions.
2. Experiments to measure the influence of axial thrust and bending moment on the strength of hybrid cross sections need to be conducted.

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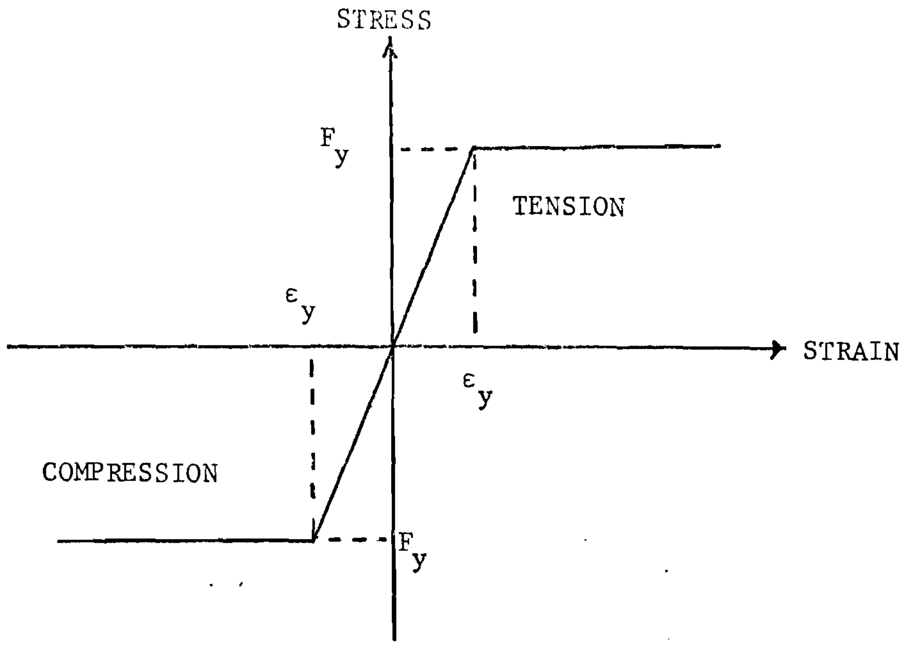
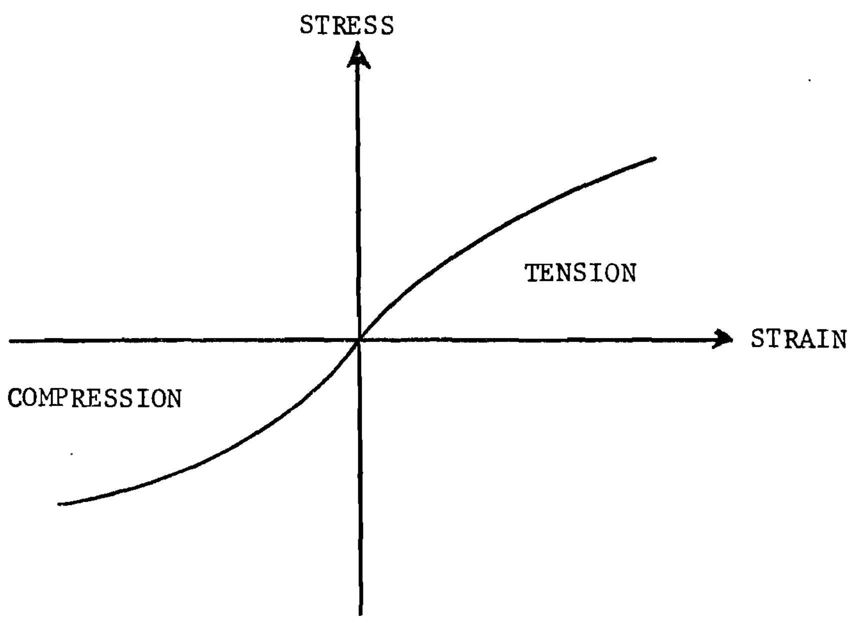
8. Samani, F., "Moment-Curvature Relation for Built-Up Steel Sections", Thesis presented to Iowa State University at Ames, Iowa, in 1968, in partial fulfillment of the requirements for the degree of Master of Science in Structural Engineering.



(a) Idealized Bilinear Stress-Strain Relationship



(b) Generalized Stress-Strain Relationship

Fig. 1 Stress-Strain Relationships.

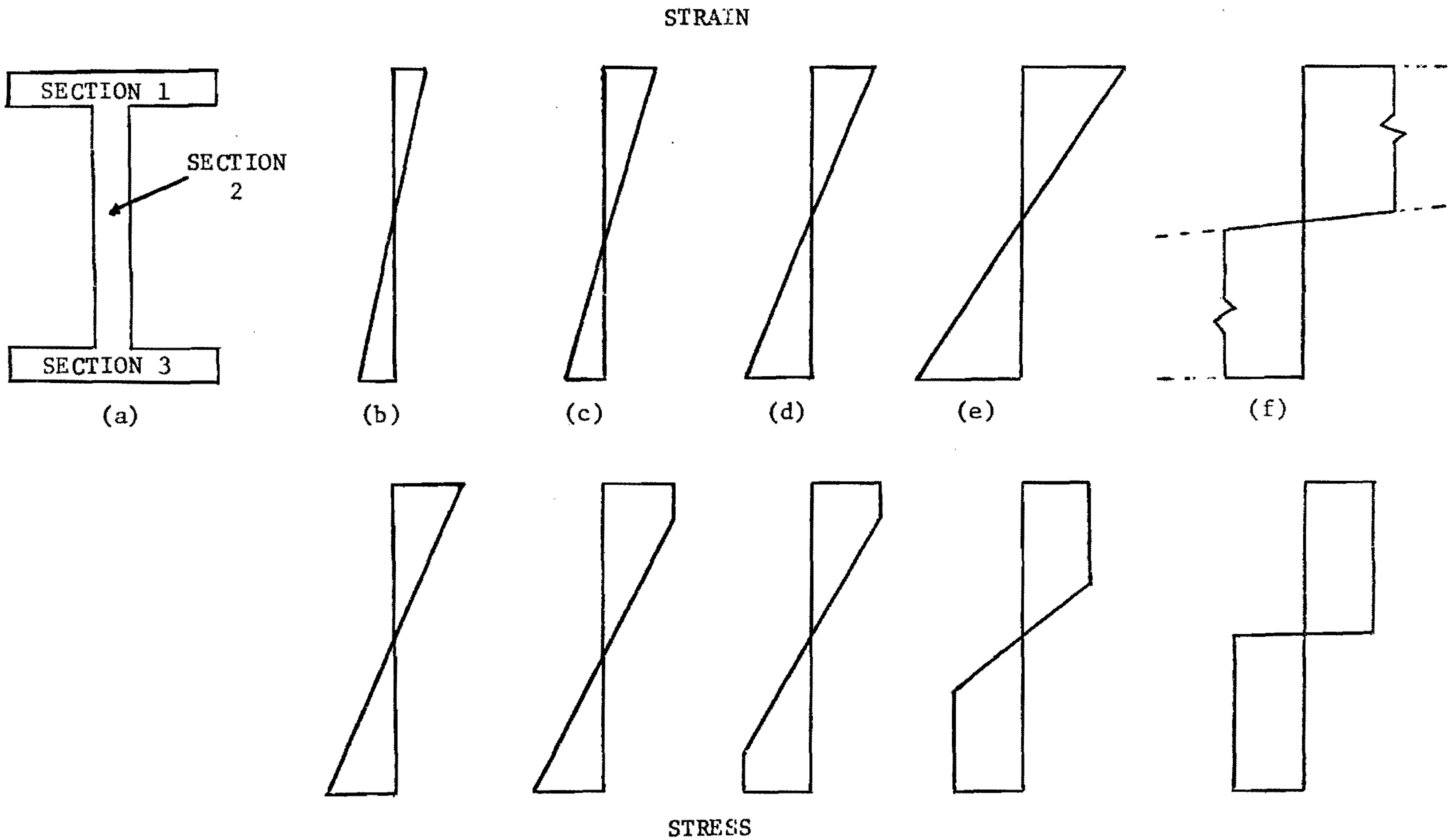


Fig. 2 Possible Strain and Stress Conditions of a Typical Wide Flange Cross Section Subject to Bending Moment.

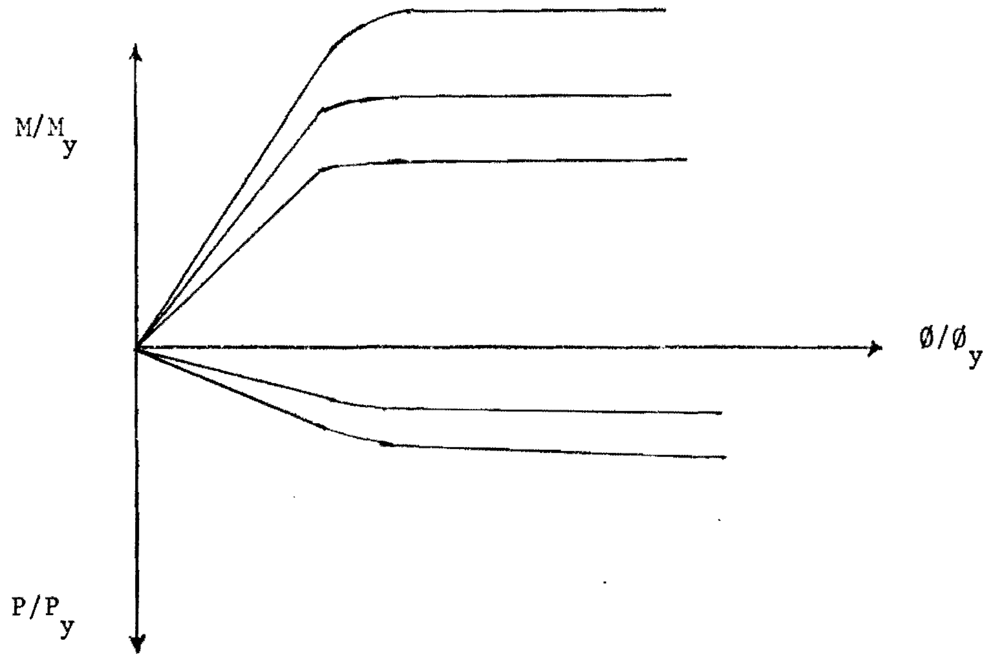


Fig. 3 Typical Auxiliary Curves.

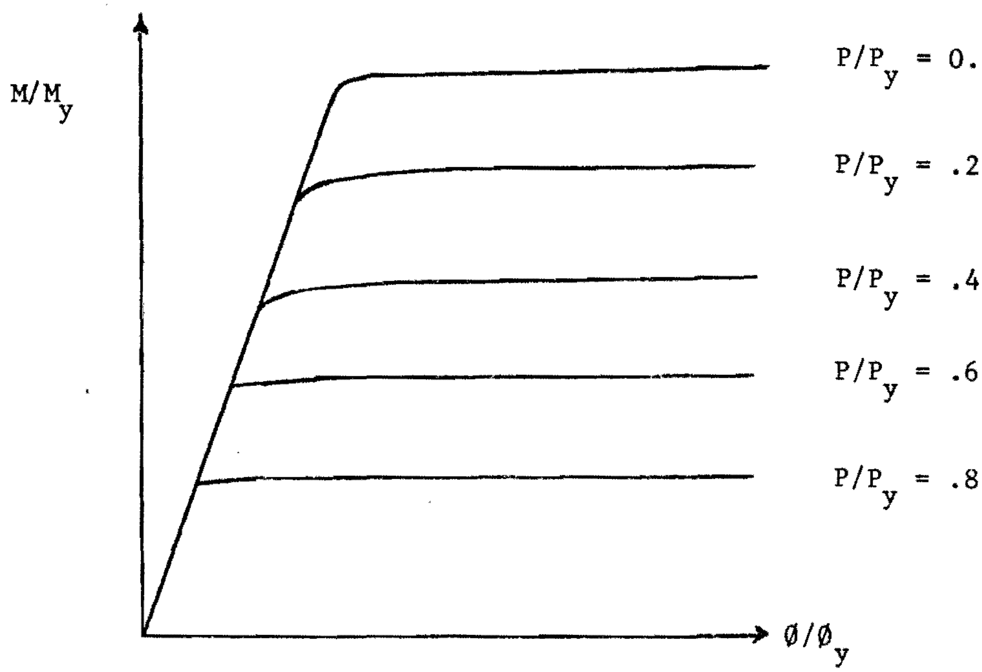
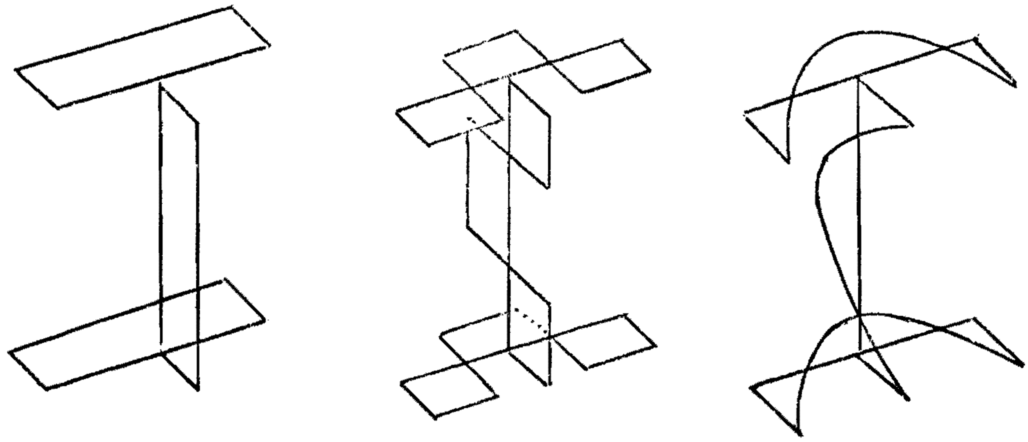


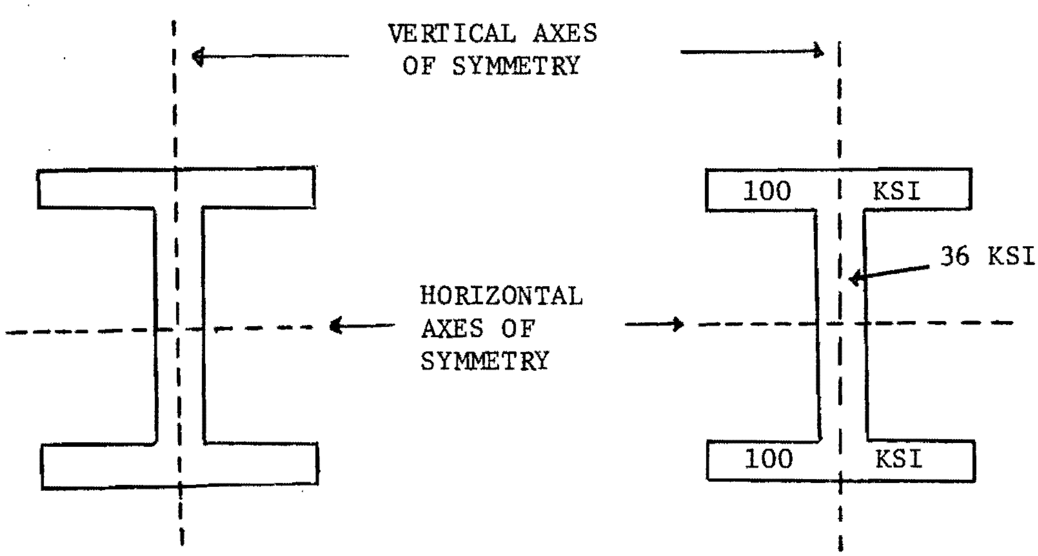
Fig. 4 Typical M-Ø-P Relationship for a Wide Flange Shape.



(a) Idealized Linear Residual Stress Patterns.

(b) Parabolic Residual Stress Pattern

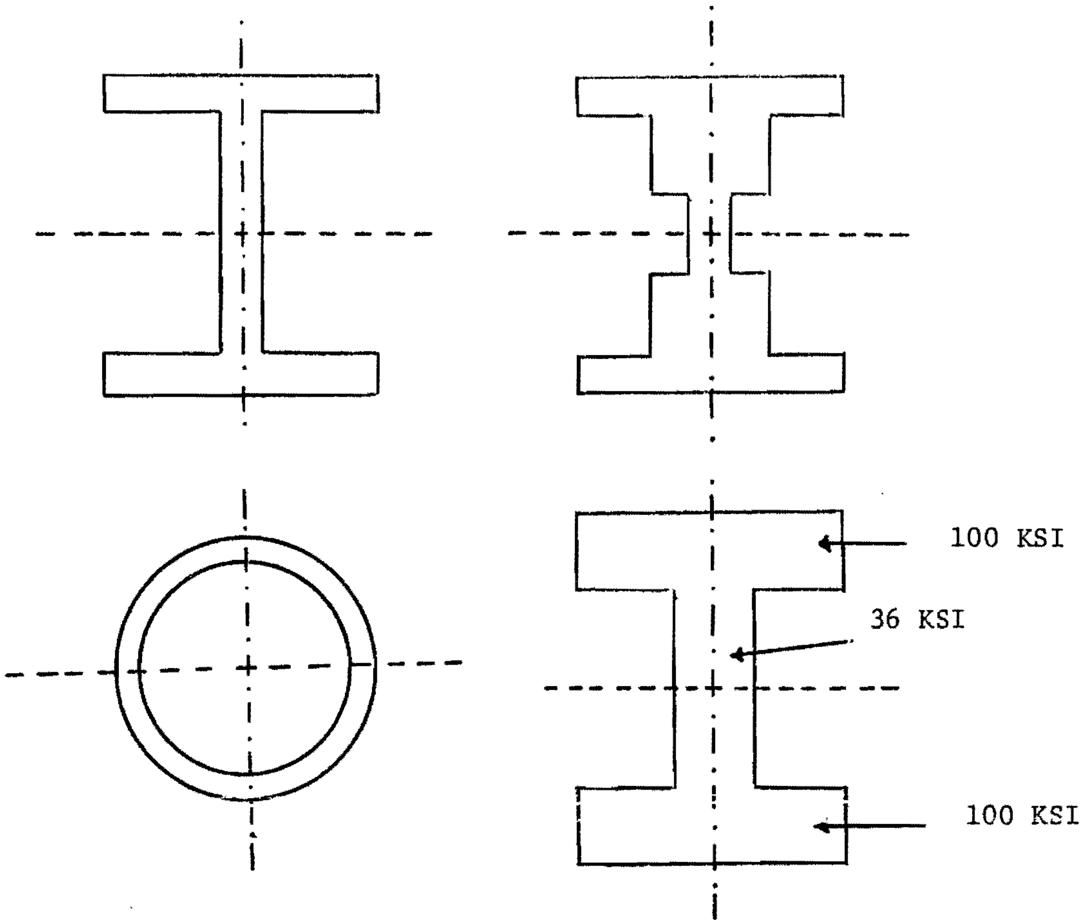
Fig. 5 Various Types of Residual Stress Patterns.



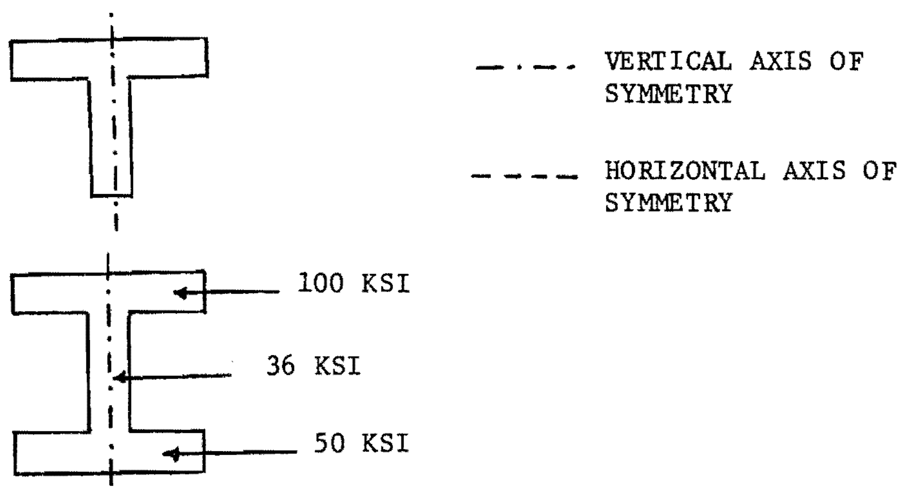
(a) Geometrically symmetric Cross Section.

(b) Materially Symmetric Cross Section.

Fig. 6 Horizontal and Vertical Symmetries.

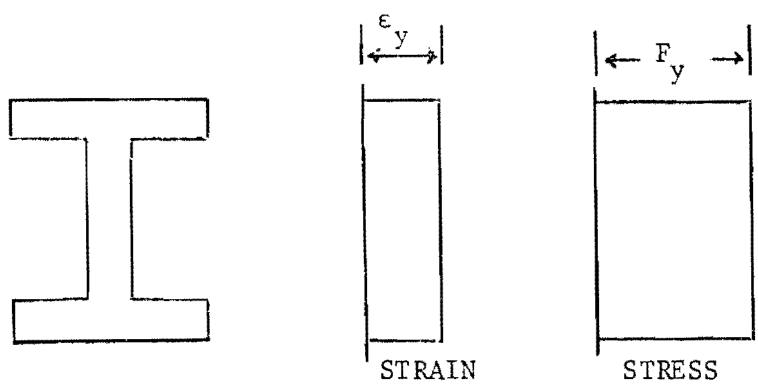


(a) Totally Symmetric Cross Sections.

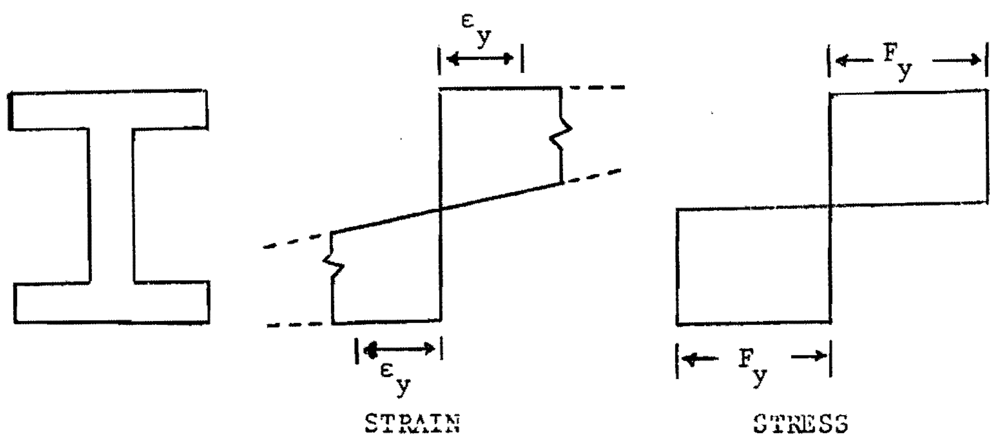


(b) Asymmetric Cross Sections.

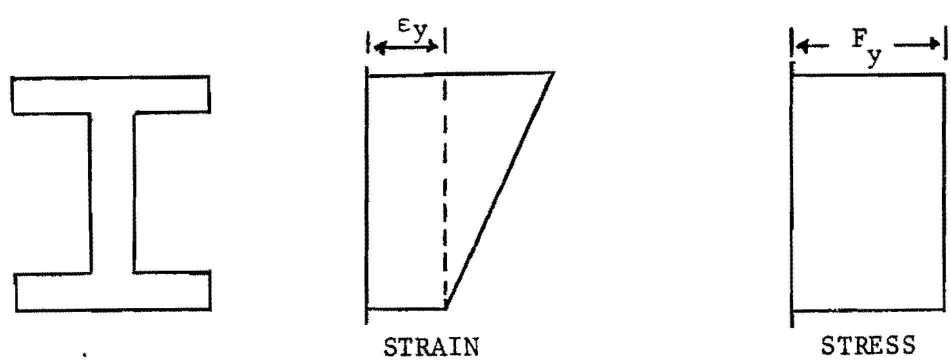
Fig. 7 Totally Symmetric and Asymmetric Cross Sections.



(a) Total Plastification Caused by Axial Thrust.



(b) Total Plastification Caused by Bending Moment.



(c) Total Plastification Caused by Bending Moment and Axial Thrust.

Fig. 8 Causes of Total Plastification.

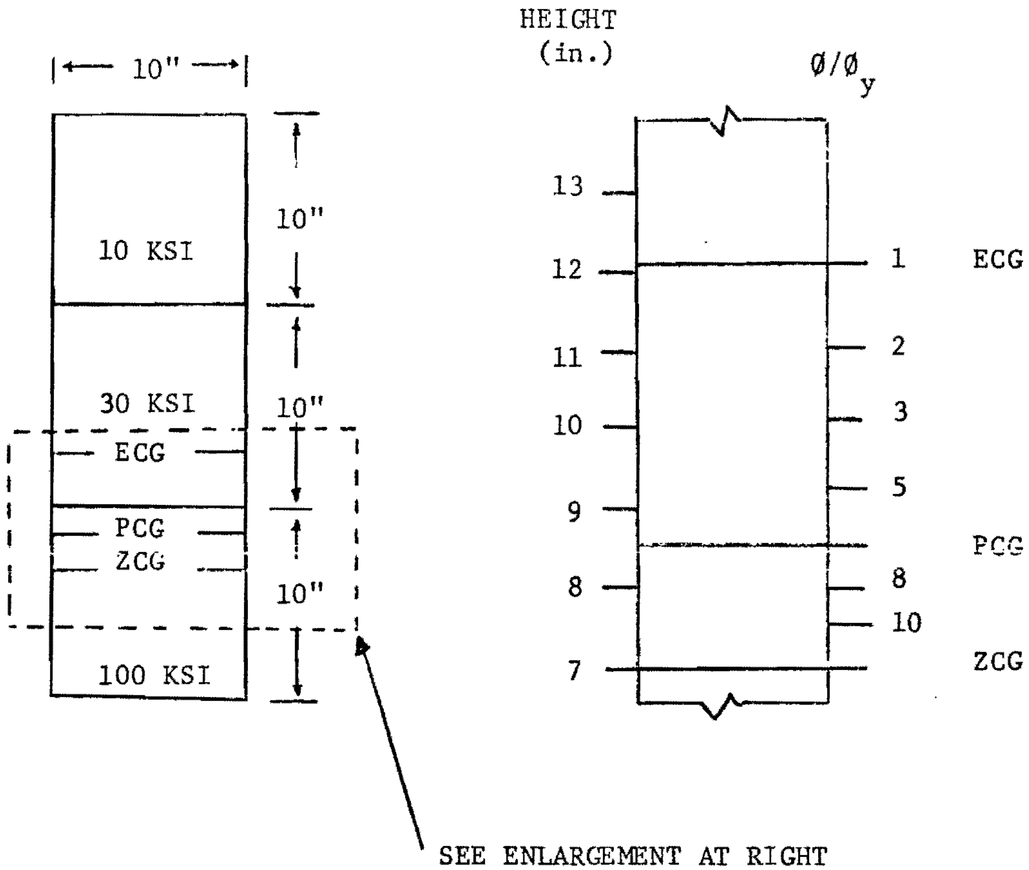
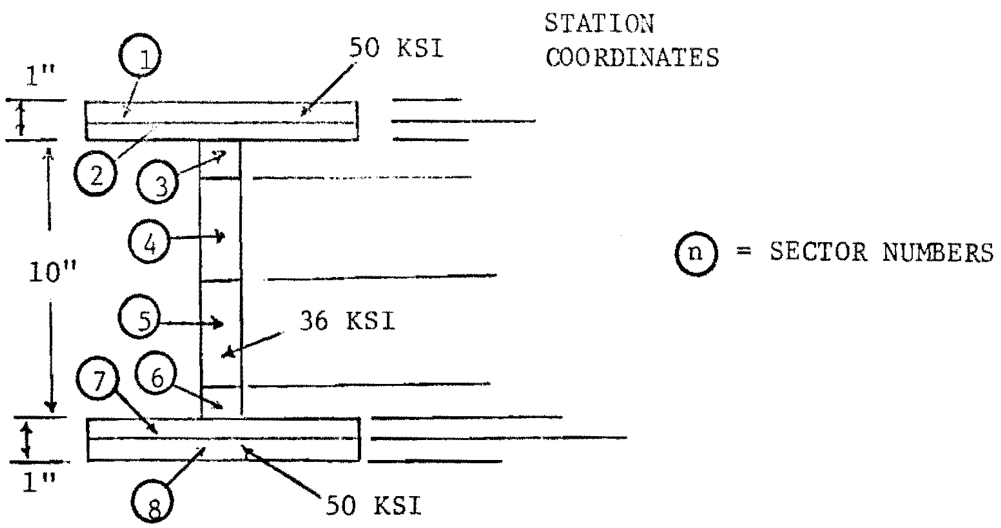
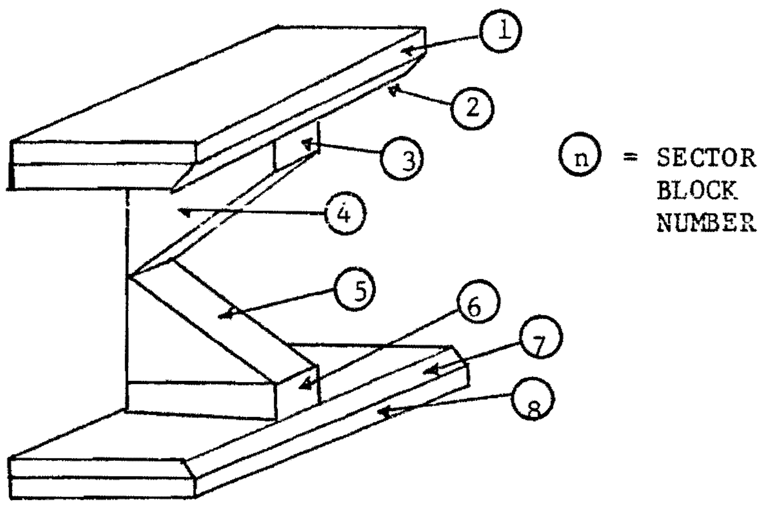


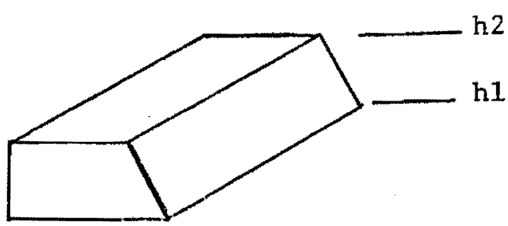
Fig. 9 Positions of the Zero Thrust Axis for Various Angles of Curvature, $\emptyset/\emptyset_y$.



(a) Possible Required Sectors and Station Coordinates.

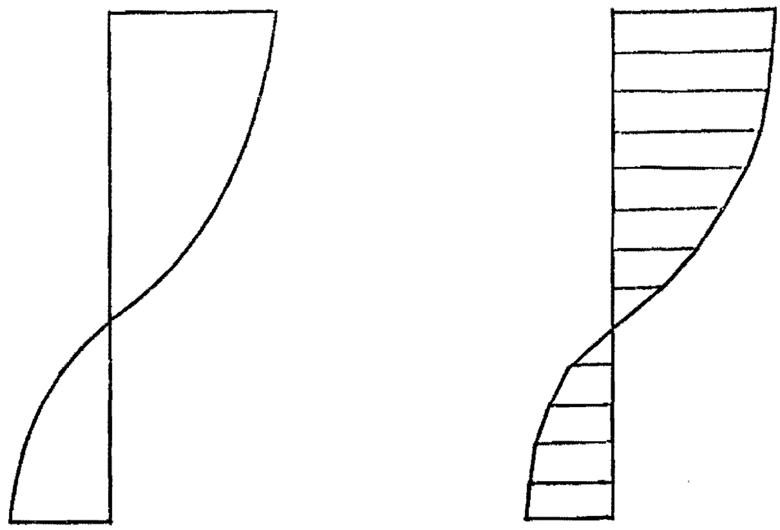


(b) Stress Block Influencing the Cross Section.



(c) Typical Finite Sector Stress Block Volume.

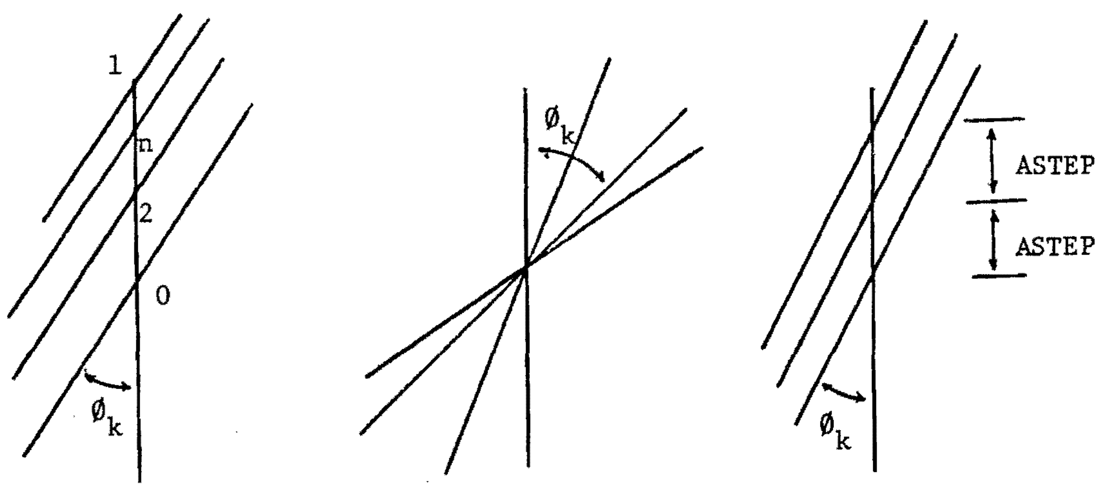
Fig. 10 Division of a Cross Section into Sectors.



(a) True Stress Distribution.

(b) Idealized Stress Distribution.

Fig. 11 Stress Distributions.



(a) Estimated Axes Positions.

(b) Generalized Angle of Curvature, ϕ_k.

(c) Axis Increments by Determined Steps.

Fig. 12 Neutral Axes for Iteration Procedures.

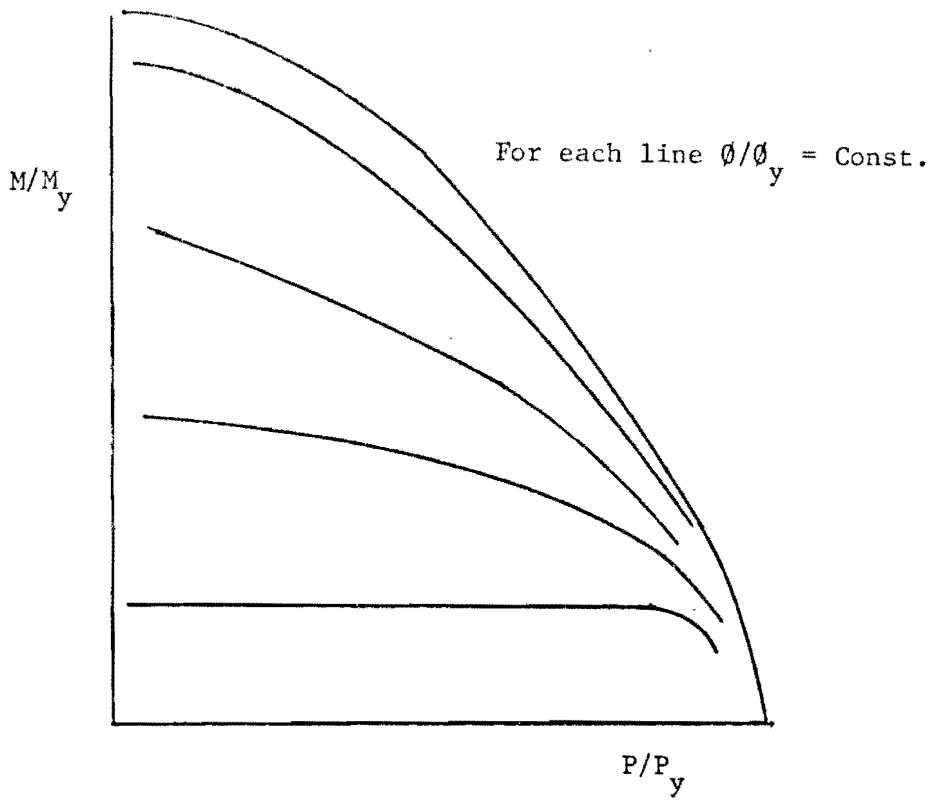


Fig. 13 Moment-Thrust Diagrams for Constant Angles of Curvature.

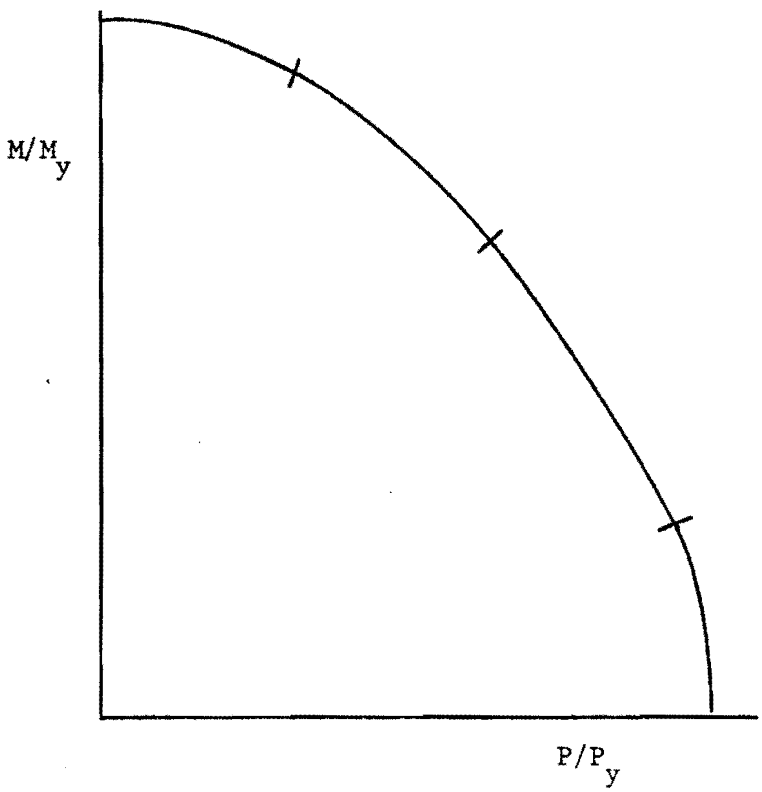


Fig. 14 Ranges of Applicability of Certain Estimation Functions.

MOMENT-CURVATURE-THRUST DATA (MEMBER 1 OF 2)

SECTION PARAMETERS

SECT. NO.	BOTTOM HEIGHT (IN.)	TOP HEIGHT (IN.)	MAXIMUM STRESS (KSI)	ELASTIC MODULUS (KSI)	SECT. WIDTH (IN.)	RESIDUAL STRESS CONSTANTS AR	BR	CR
1	0.000	0.375	100.0	30000.0	4.000	0.000E 00	0.000E 00	0.000E 00
2	0.375	6.375	57.5	30000.0	0.187	0.000E 00	0.000E 00	0.000E 00
3	6.375	6.750	100.0	30000.0	4.000	0.000E 00	0.000E 00	0.000E 00

MATERIAL NUMBER	RESIDUAL STRESSES	NON-BILINEAR STRESS-STRAIN	SECTION DIVIDER
1	0	1	4.000
0	0	0	0.000
0	0	0	0.000

NONBILINEAR STRESS-STRAIN FUNCTIONS

MATERIAL NO. 1 --- 4 LINEAR APPROXIMATIONS ---DIVIDING STRAIN= 0.383E-03

STRESSES=	-.103E 03	-.100E 03	0.000E 00	0.100E 03	0.103E 03
STRAINS =	-.766E-02	-.383E-02	0.000E 00	0.383E-02	0.766E-02

SHAPE FACTOR = 0.162150E 01
THRUST P(Y) = 0.364687E 03 K
MOMENT M(Y) = 0.649570E 03 K-IN.
ANGLE PHIY = 0.638888E-03 RAD.
GRAVITY CENTER= 0.337500E 01 IN.

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=0.	M/M(Y) FOR P/P(Y)=.2	M/M(Y) FOR P/P(Y)=.4	M/M(Y) FOR P/P(Y)=.6	M/M(Y) FOR P/P(Y)=.8	M/M(Y) FOR P/P(Y)=1.0
1.000	1.6476	1.5015	1.1367	0.8133	0.4712	0.0000
1.250	1.7067	1.5128	1.1718	0.8218	0.4732	0.0000
1.500	1.7167	1.5254	1.1738	0.8276	0.4809	0.0000

Fig. 15 General Printout Format.

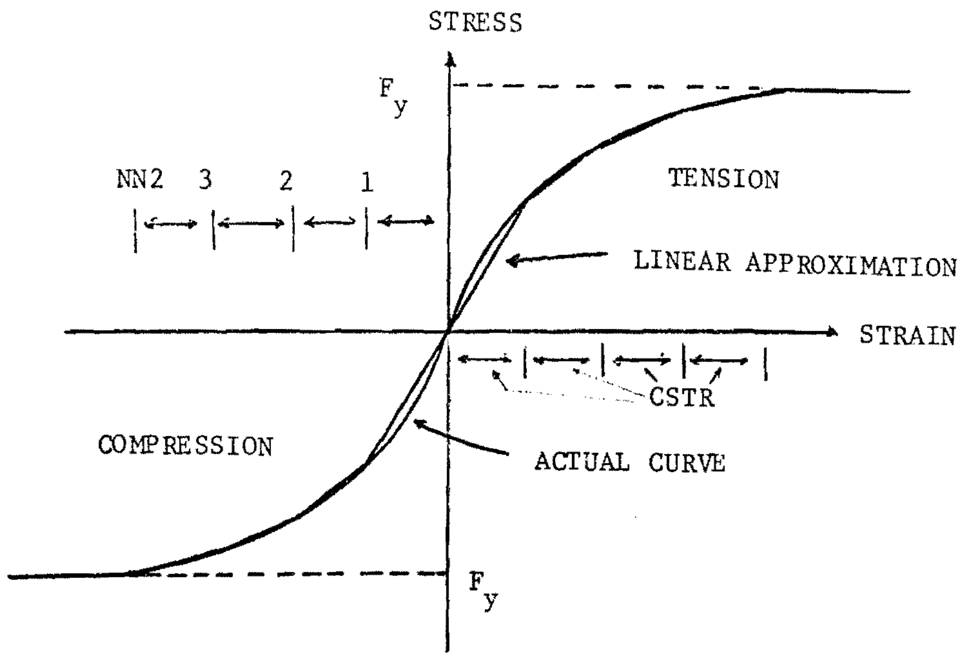


Fig. 16 Nonbilinear Stress-Strain Approximations.

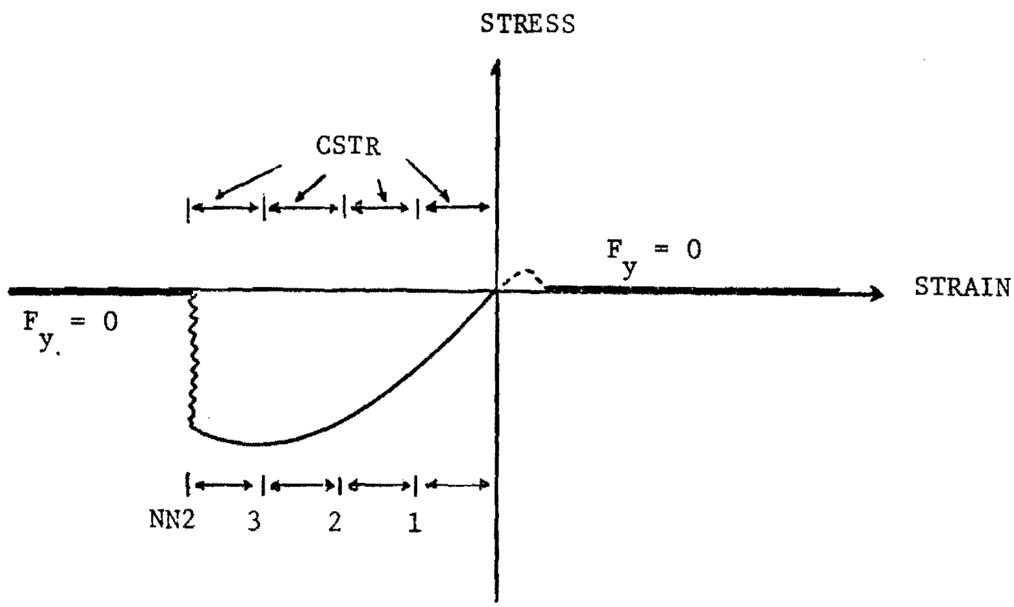


Fig. 17 Stress-Strain Approximation for Concrete.

```
SHAPE FACTOR    =  0.11065E 01
THRUST P(Y)     =  0.32337E 03   K
MOMENT M(Y)     =  0.97195E 03   K-IN.
ANGLE PHIY      =  0.30000E-03   RAD.
GRAVITY CENTER= 0.40000E 01   IN.
```

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=0.	M/M(Y) FOR P/P(Y)=.2	M/M(Y) FOR P/P(Y)=.4
0.250	0.2449	0.2449	0.2449
0.500	0.4999	0.4999	0.4994
0.750	0.7500	0.7323	0.6598
1.000	1.0000	0.8799	0.6938
2.000	1.0923	0.9648	0.7364
3.000	1.1002	0.9870	0.7472
4.000	1.1030	0.9943	0.7517
5.000	1.1043	0.9982	0.7541

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=.6	M/M(Y) FOR P/P(Y)=.8	M/M(Y) FOR P/P(Y)=1.0
0.250	0.2499	0.2202	0.0000
0.500	0.4401	0.2412	0.0000
0.750	0.4695	0.2467	0.0000
1.000	0.4824	0.2490	0.0000
2.000	0.4981	0.2519	0.0000
3.000	0.5020	0.2529	0.0000
4.000	0.5038	0.2535	0.0000
5.000	0.5050	0.2541	0.0000

Fig. 18 Data for a W 8X31

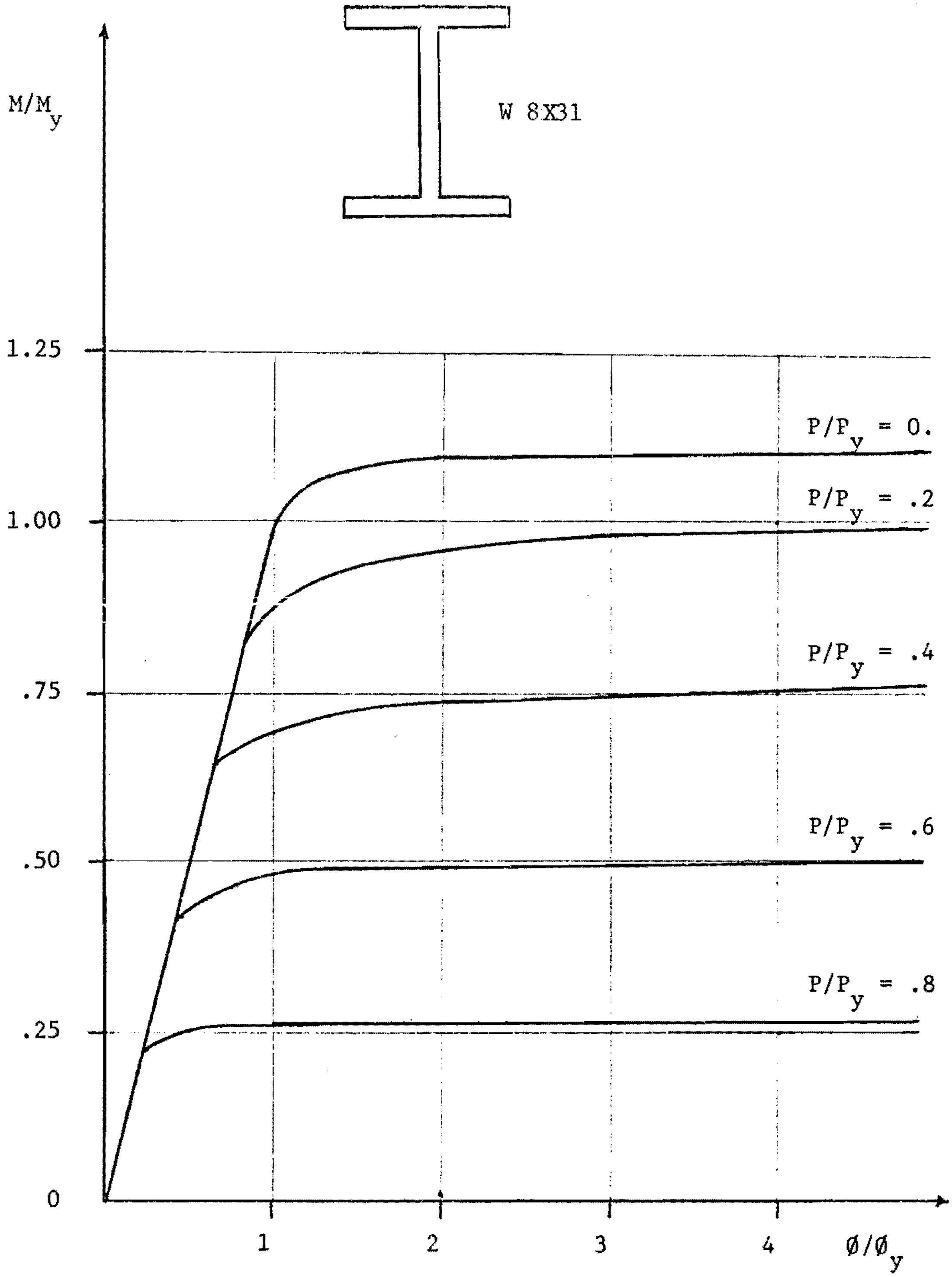


Fig. 19 M-ϕ-P Diagram for a W 8X31.

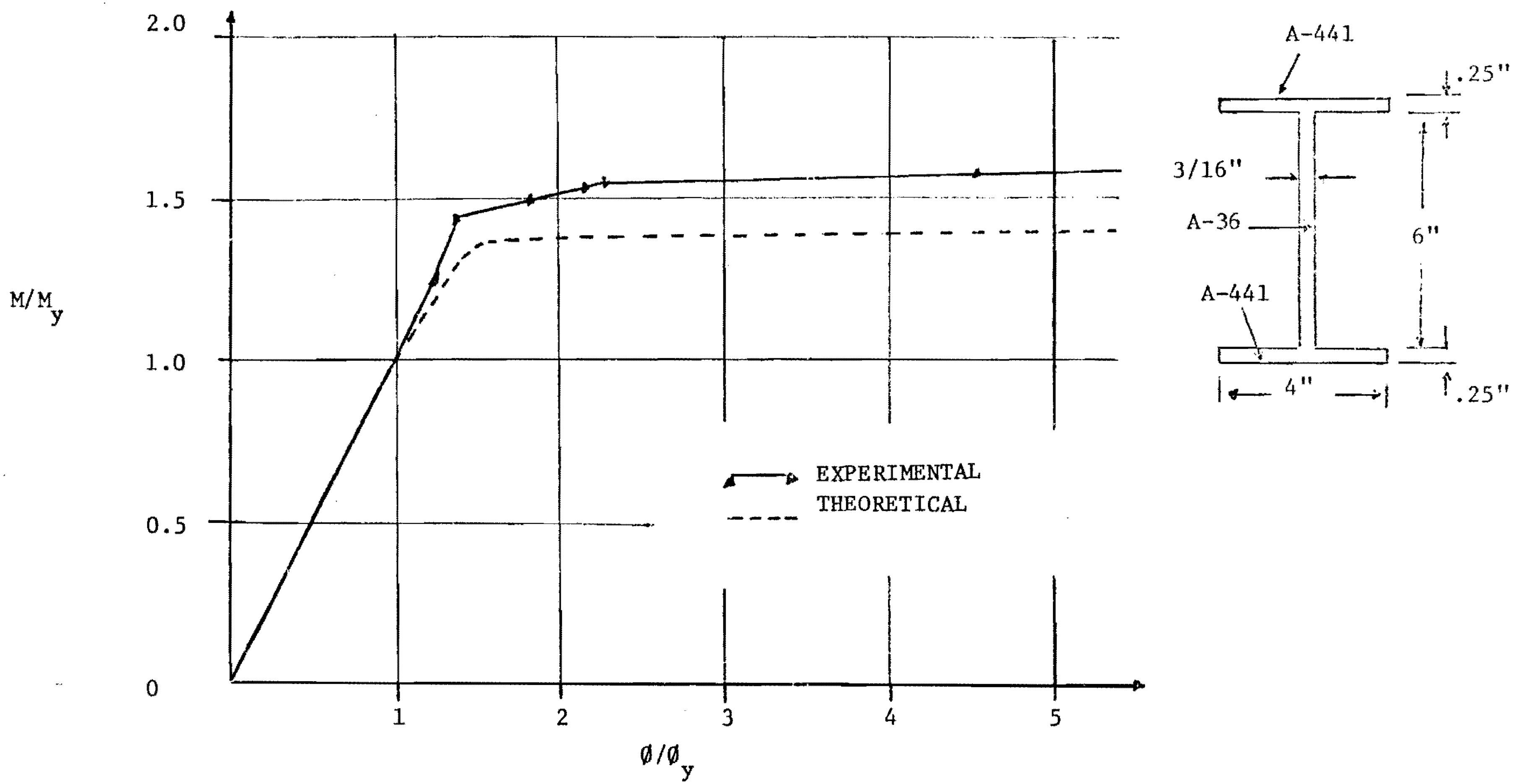


Fig. 20 Experimental and Theoretical M-Ø Relationships for Cross Section T1.

SHAPE FACTOR = 0.139704E 01
THRUST P(Y) = 0.160000E 03 K
MOMENT M(Y) = 0.305555E 03 K-IN.
ANGLE PHIY = 0.444444E-03 RAD.
GRAVITY CENTER= 0.325000E 01 IN.

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=0.	M/M(Y) FOR P/P(Y)=.2	M/M(Y) FOR P/P(Y)=.4
0.250	0.2500	0.2499	0.2499
0.500	0.5000	0.5000	0.4998
0.750	0.7500	0.7499	0.7398
1.000	1.0000	0.9943	0.8759
1.500	1.3643	1.1712	0.9216
2.000	1.3786	1.2172	0.9429
3.000	1.3888	1.2620	0.9618
4.000	1.3924	1.2784	0.9699
5.000	1.3941	1.2818	0.9741

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=.6	M/M(Y) FOR P/P(Y)=.8	M/M(Y) FOR P/P(Y)=0.
0.250	0.2498	0.2240	0.0000
0.500	0.4853	0.3228	0.0000
0.750	0.6090	0.3251	0.0000
1.000	0.6246	0.3261	0.0000
1.500	0.6387	0.3269	0.0000
2.000	0.6449	0.3273	0.0000
3.000	0.6502	0.3278	0.0000
4.000	0.6525	0.3281	0.0000
5.000	0.6583	0.3283	0.0000

Fig. 21 Data Printout for Cross Section T1.

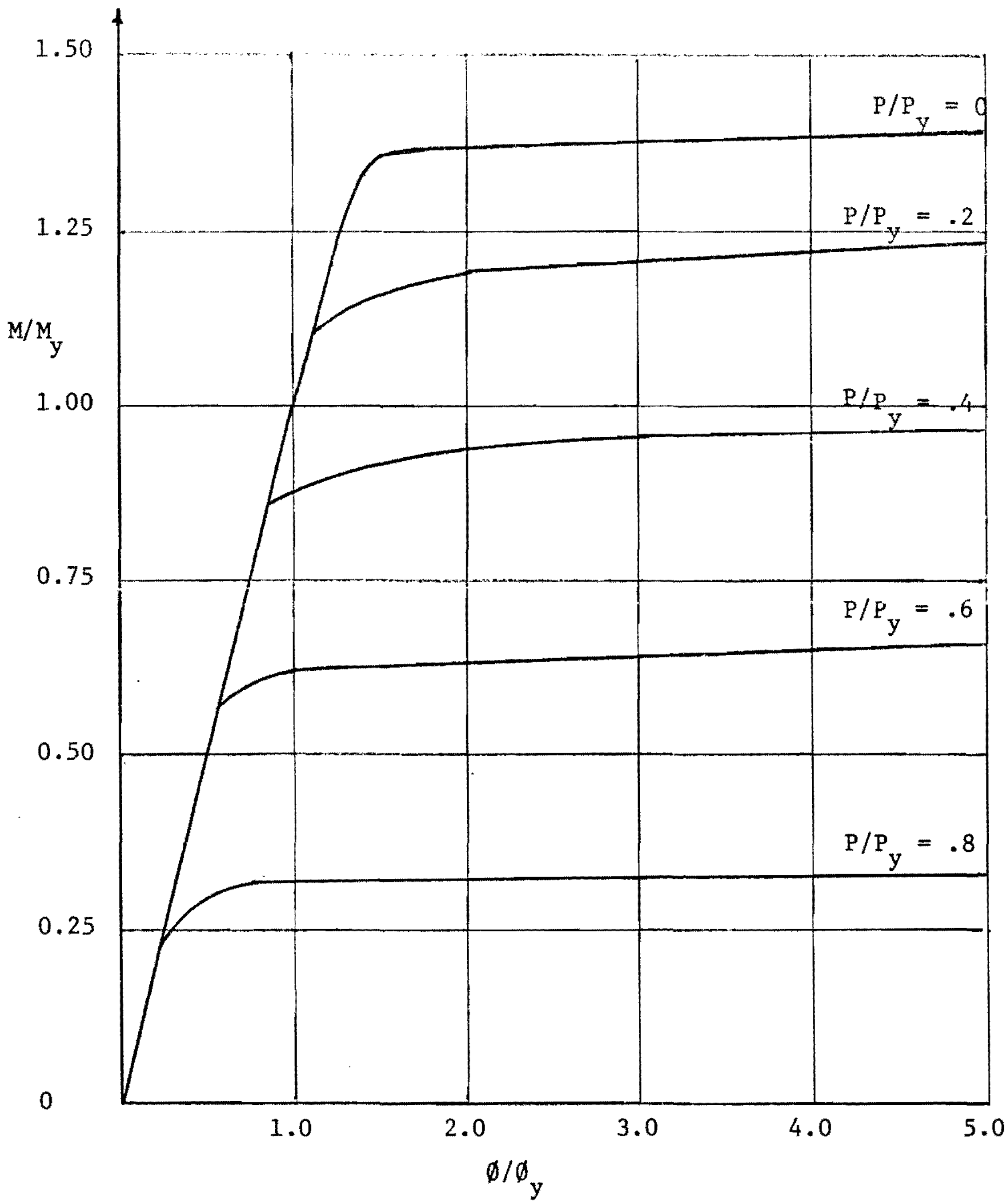


Fig. 22 M-Ø-P Diagram for Cross Section T1.

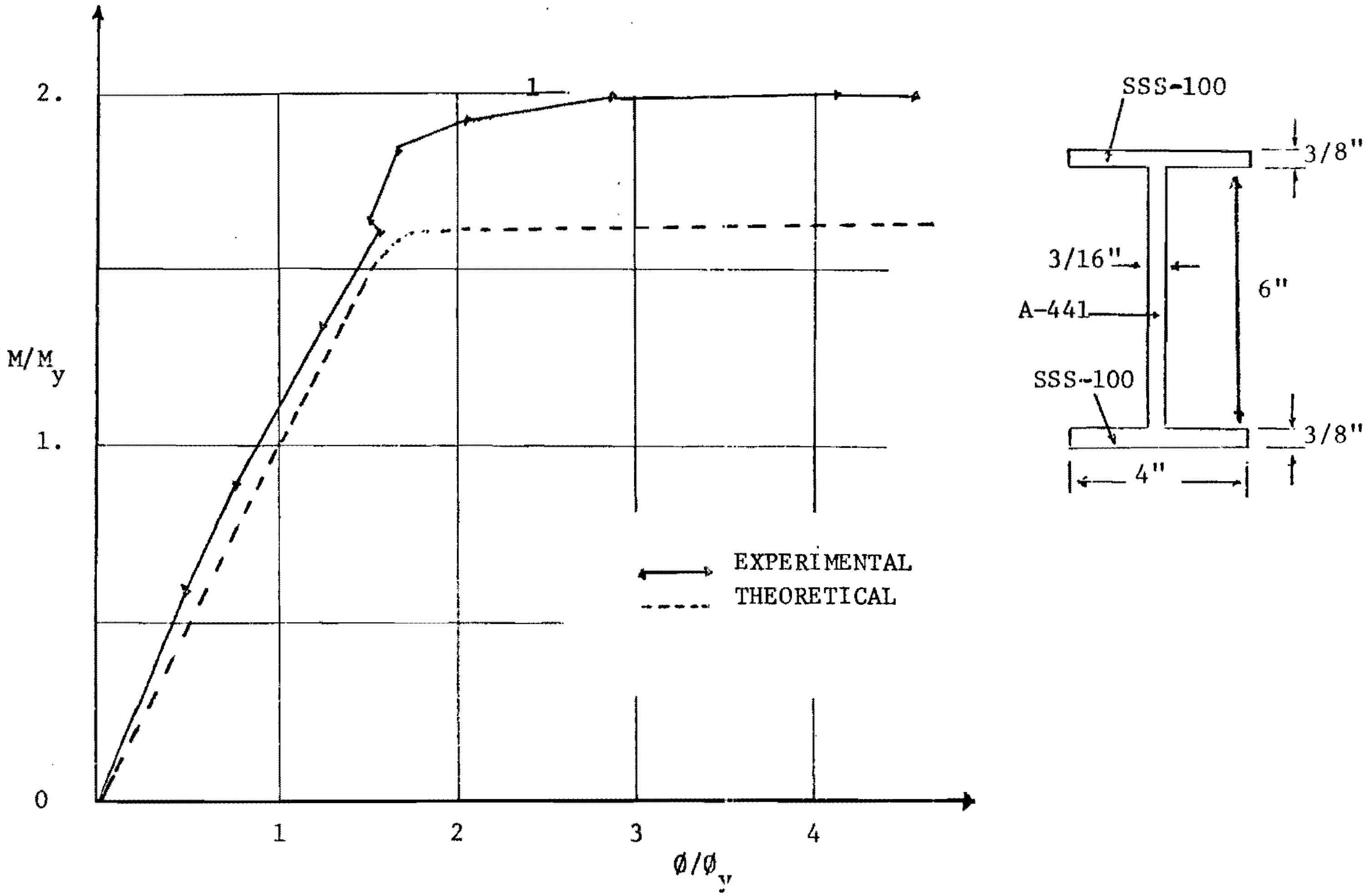


Fig. 23 Experimental and Theoretical M-Ø Relationships for Cross Section T2.

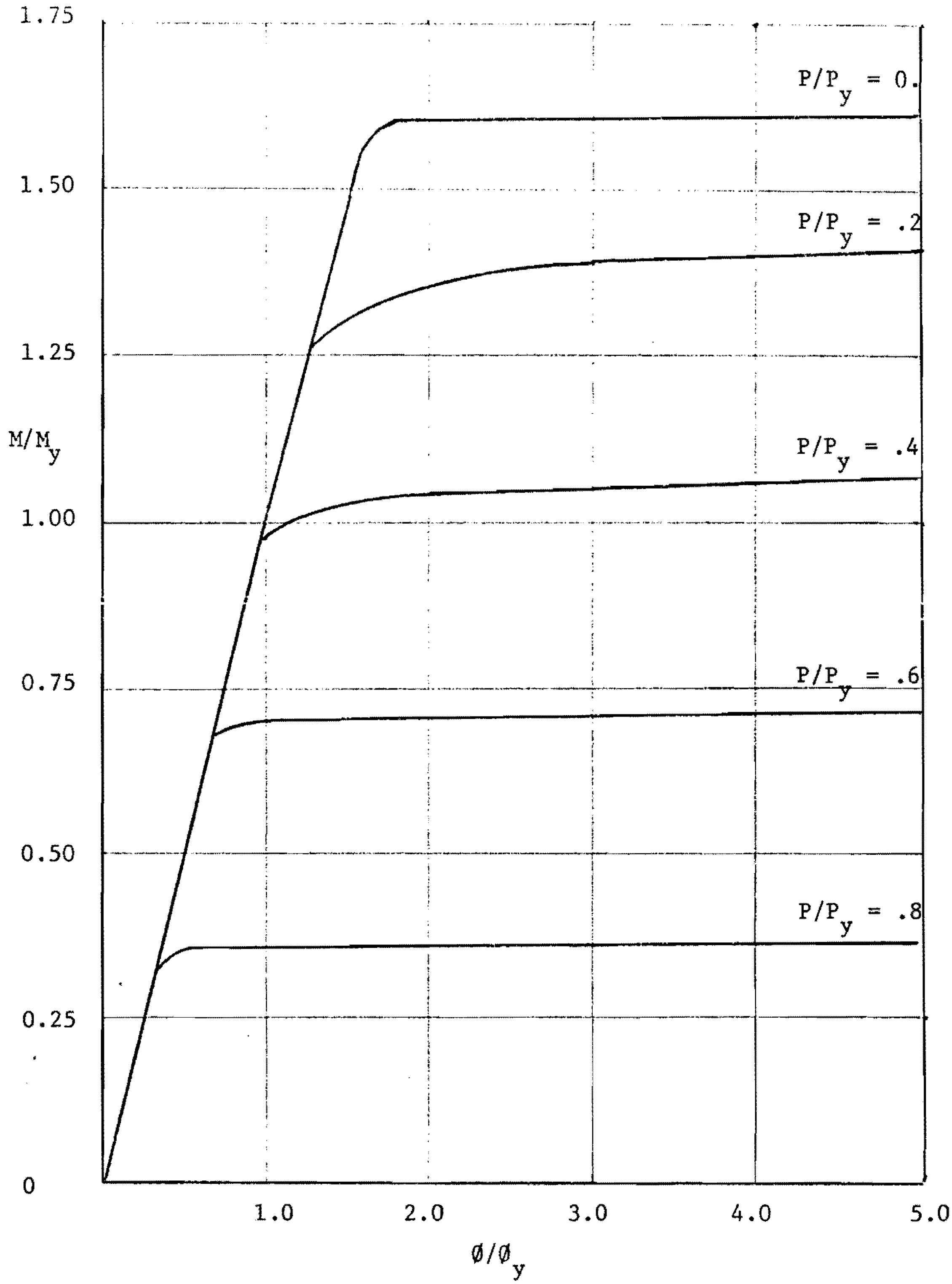


Fig. 24 M-Ø-P Diagram for Cross Section T2.

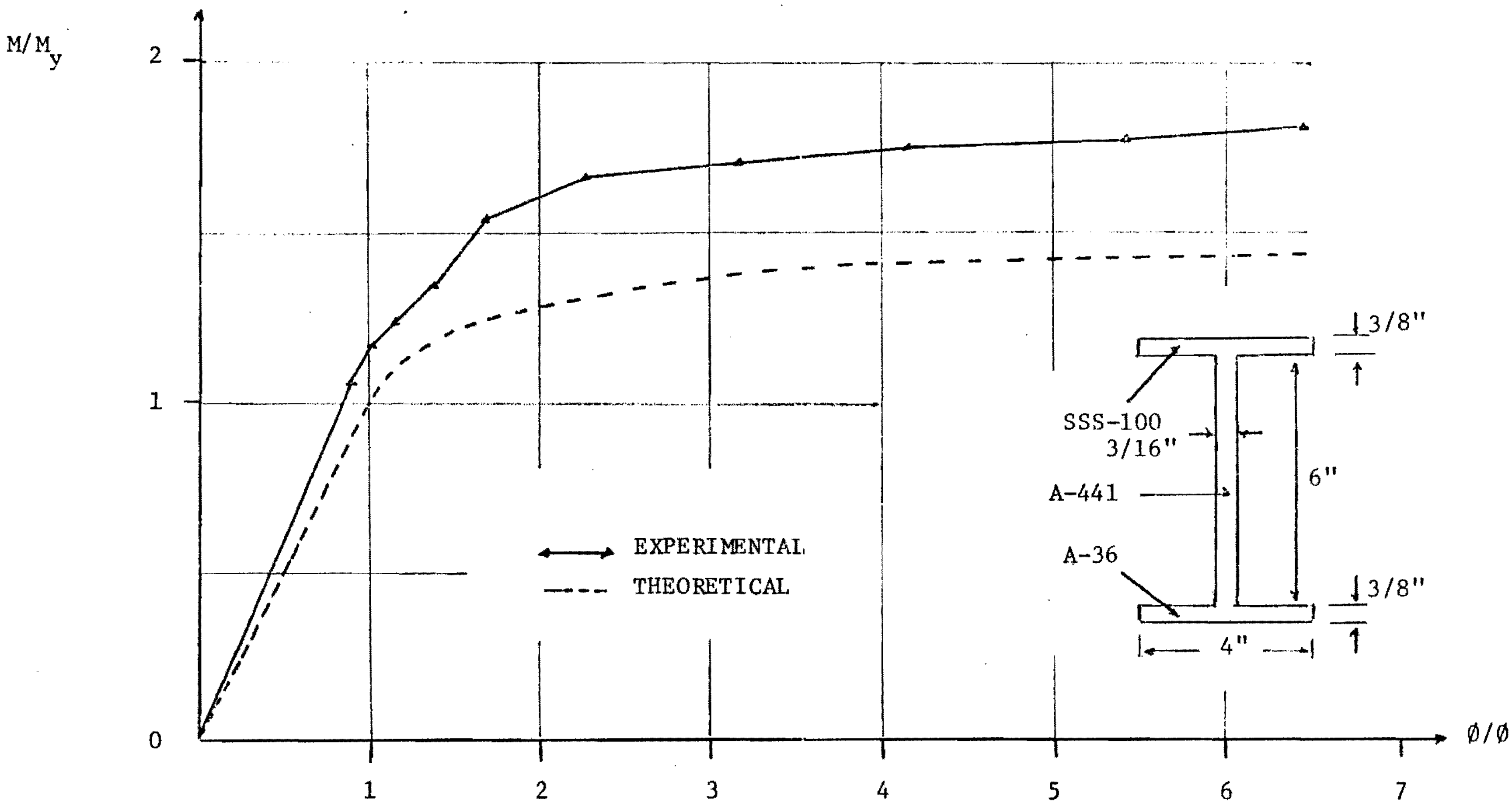
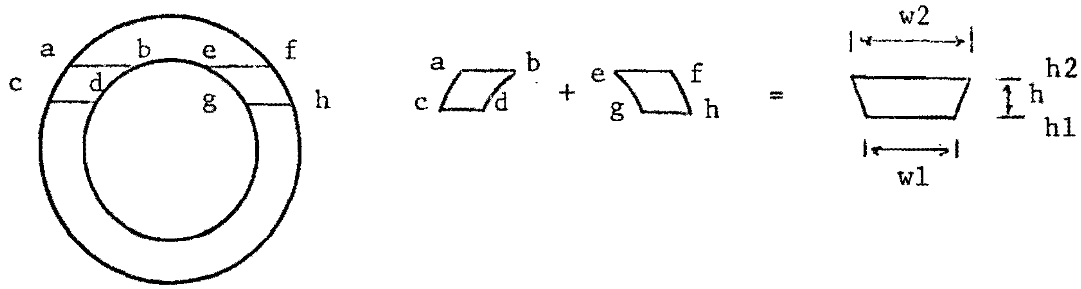
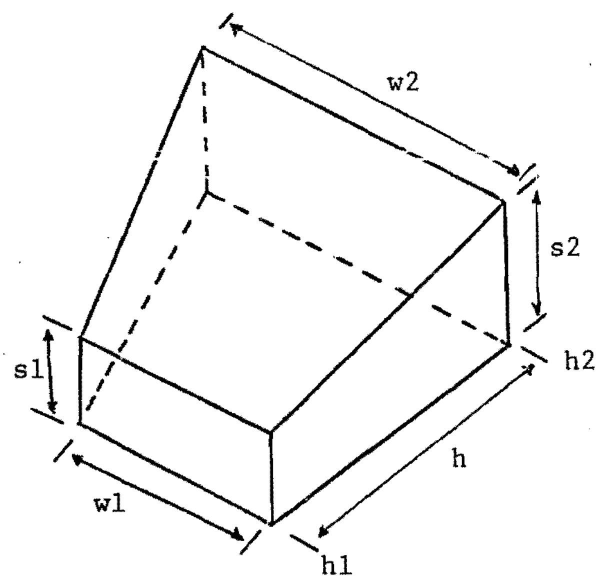


Fig. 25 Experimental and Theoretical M-$\emptyset$ Relationships for cross section T3.



(a) Circular Sector Formation.



(b) Generalized Stress Block Volume.

Fig. 26 Variations for Circular Tubes.

STRUCTURAL TUBE MOMENT-CURVATURE-THRUST DATA

OUTER RADIUS	=	0.700000E 01	IN.
INNER RADIUS	=	0.650000E 01	IN.
YIELD(MAXIMUM)STRESS	=	0.360000E 02	KSI
MODULUS OF ELASTICITY	=	0.300000E 05	KSI
SECTOR HEIGHT	=	0.400000E 00	IN.
SHAPE FACTOR	=	0.131918E 01	
STRAIN AXIS INCREMENT	=	0.107692E 01	IN.
THRUST P(Y)	=	0.763406E 03	K
MOMENT M(Y)	=	0.248788E 04	K-IN.
ANGLE PHI(Y)	=	0.171428E-03	RAD.

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=0.	M/M(Y) FOR P/P(Y)=.2	M/M(Y) FOR P/P(Y)=.4
0.250	0.2489	0.2478	0.2467
0.500	0.4978	0.4967	0.4967
0.750	0.7467	0.7460	0.7080
1.000	0.9956	0.9451	0.8111
2.000	1.2563	1.1861	0.9807
3.000	1.2913	1.2239	1.0292
4.000	1.3032	1.2365	1.0449
5.000	1.3086	1.2423	1.0520

PHI/PHI(Y)	M/M(Y) FOR P/P(Y)=.6	M/M(Y) FOR P/P(Y)=.8	M/M(Y) FOR P/P(Y)=1.0
0.250	0.2456	0.2318	0.0000
0.500	0.4696	0.3037	0.0000
0.750	0.5627	0.3301	0.0000
1.000	0.6112	0.3446	0.0000
2.000	0.6924	0.3696	0.0000
3.000	0.7260	0.3802	0.0000
4.000	0.7423	0.3853	0.0000
5.0000	0.7536	0.3894	0.0000

Fig. 27 M-Ø-P Data for a 14.0 in. O.D., .5 in. Wall Thickness Standard Tube.

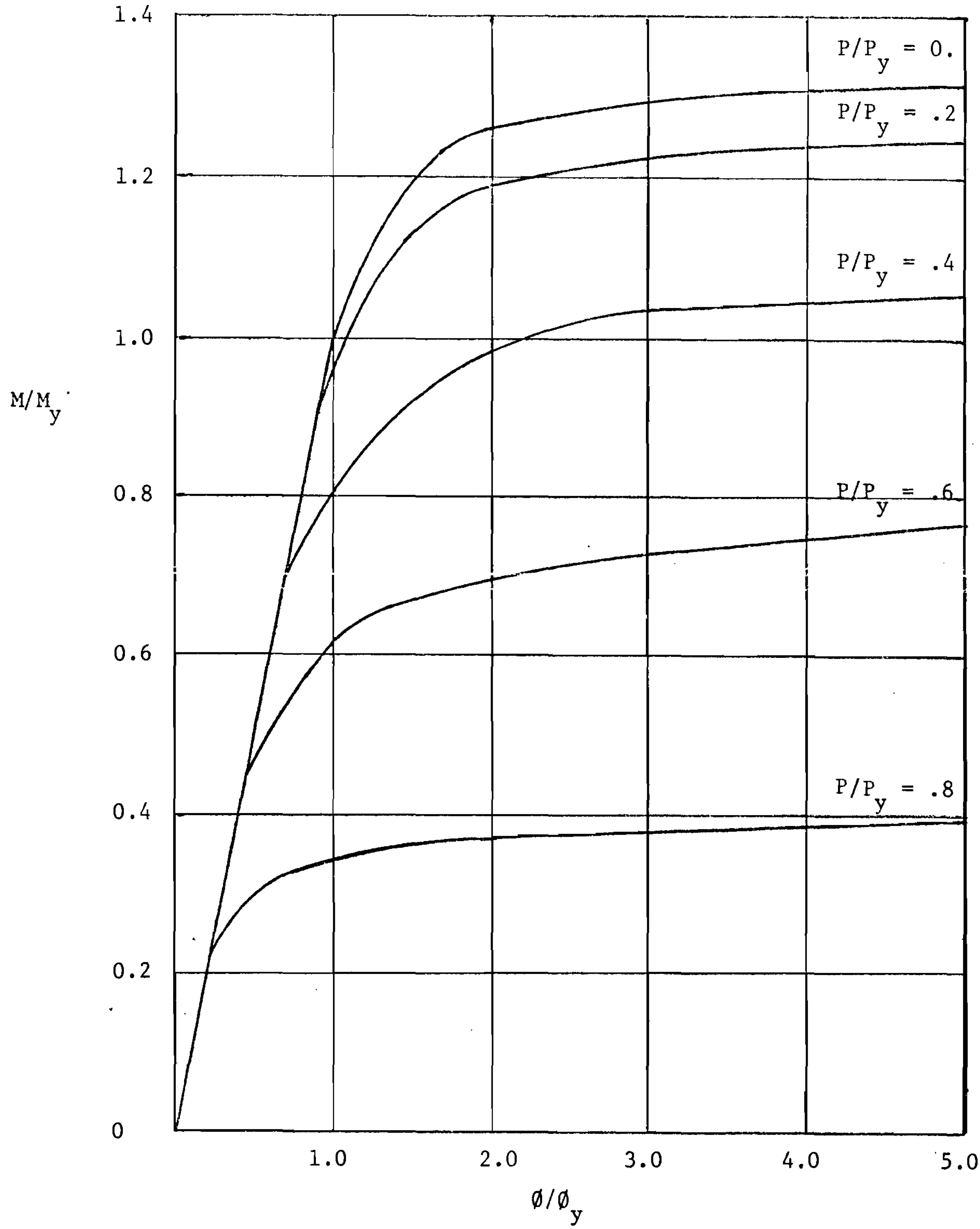
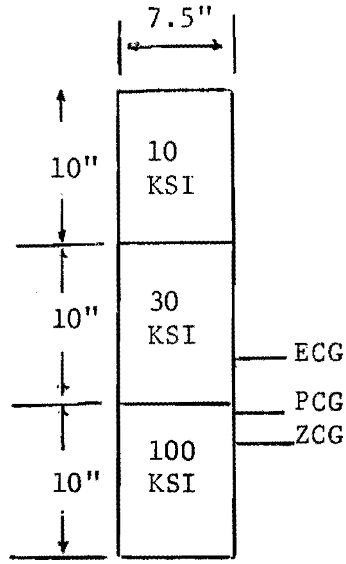
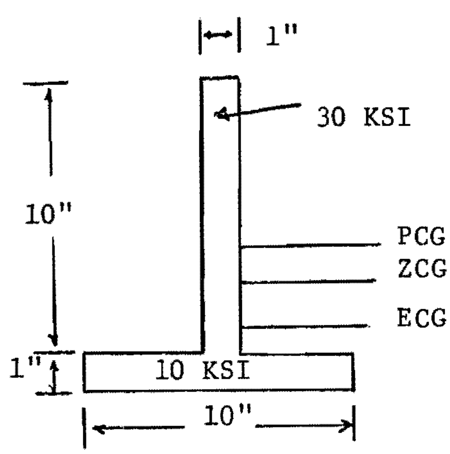


Fig. 28 M-ϕ-P Diagram for a 14.0 in. OD., 0.5 in. Wall Thickness Standard Tube.



(a) PCG between ECG and ZCG.



(b) PCG not Between ECG and ZCG.

Fig. 29 Possible Combinations of PCG, ECG and ZCG.

Table 1. Values of Successive Iterations for $F(X)=e^{-x}$

P=3.5		$F(X)=e^{-x}$	Degree 1	Degree 2	Degree 3	Degree 4
	x	1	2	3	4	5
1	0.0	1.0	-	-	-	-
2	0.1	0.904837	0.762093	-	-	-
3	0.2	0.818731	0.773414	0.779074	-	-
4	0.3	0.740818	0.784015	0.778534	0.778804	-
5	0.4	0.670320	0.793950	0.778021	0.778811	0.778801

Table 2. Values of Successive Iterations for $F(X)=x^2$

P=3.5		$F(X)=x^2$	Degree 1	Degree 2	Degree 3	Degree 4
	x	1	2	3	4	5
1	1.0	1.0	-	-	-	-
2	2.5	6.25	9.75	-	-	-
3	3.0	9.0	11.0	12.25	-	-
4	4.0	16.0	13.5	12.25	-	-
5	4.5	20.25	14.75	12.25	-	-

APPENDIX A

For asymmetric cross sections the proper moment center is the elastic centroid when elastic stress conditions govern. For total plasticity of the type shown in Fig. 8c the plastic centroid must be used. Since all angles of curvature eventually yield this type (Fig. 8c) of strain relationship, the final moment center for any cross section is always the plastic centroid.

For any angle of curvature the initial moment center can be assigned to the zero thrust axis and the final moment center to the plastic centroid. This change in moment center suggests a moment center shift from the zero thrust axis to the plastic centroid when the cross section is totally plastified. That this axis shift occurs only when the cross section becomes totally plastified is unlikely, and a more uniform shift as total plasticity is reached is more reasonable.

As previously mentioned, the moment center shift is not defined and, therefore, only pure bending cases are treated. However, one further solution is possible.

If the zero thrust axis initially lies on the plastic centroid, no moment center shift is possible and calculations for all thrust percentages can be made. The only criterion is that the PCG must lis between the ECG and ZCG of the cross section. Figure 29 shows that the PCG does not always lie in such a position.

Asymmetric cross sections whose plastic centroid lies between the ECG and ZCG can be treated for bending moment and any thrust percentage when the angle of curvature whose zero thrust axis lies on the plastic centroid is used. If the PCG does not lie between the ECG and ZCG only pure bending can be treated.

The angle of curvature, $\emptyset_s$, whose zero thrust axis lies on the plastic centroid, greatly effects the usability of the thrust calculations. If $\emptyset_s$ is large, all thrust percentages will ies on straight line portions of the M-$\emptyset$-P diagrams. Since such values are of little use,these calculations are useful only if $\emptyset_s$ is small (less than 2). M-$\emptyset$-P values for $\emptyset/\emptyset_y > 2$ are really useless because of local and lateral buckling.

APPENDIX B: AITKEN'S PROCESS

Aitken's process is a linear interpolation procedure that does not require a uniform grid. Table 1 gives values of $F(X) = e^{-X}$ over a uniform grid. Table 2 gives $F(X) = x^2$ over a nonuniform grid. Using Aitken's process, estimations for an unknown F(X) can be obtained by a succession of second order determinates:

$$Y(J,I) = \frac{\begin{vmatrix} Y(L,L) & X(J)-P \\ Y(J,L) & X(L)-P \end{vmatrix}}{X(J) - X(L)}$$

where P is the point for which F(X) is to be evaluated, I and J are the column and row of the estimation matrix, [Y], respectively, and L denoted the column previous to I.

The process is applied to [Y] until a convergence occurs, as in the third column of [Y], Table 2. The convergent value is usually a very good approximation to the true value, but since no estimation of the truncation error is available, the derived value should not be accepted as exact without further investigation.

Table 1 indicates that e^{-X} may be closely approximated by a third order equation over the given interval as a fairly good convergence is obtained in the fourth column of [Y]. The fifth column produces e^{-X} to the same accuracy as tabulated values. Note that the X^2 value (Table 2) converges in the third column of [Y] as expected (second order curve), since the degree of the function is known. After the first convergence is obtained, successive iteration columns yield identical results, thus continuing the process is unwarranted.

Subroutine AITKN contains the basic Aitken's process calculation. The order of operation is:

(1) A check is made for an exact fit of P in the data for a given function. P is any value for which F(P) is to be determined.

(2) If no exact fit is found the point in the table nearest to where P should lie is determined. This search guarantees that points used in the iteration will be close to and lie on each side of the unknown F(P).

(3) Checks are made to varify that no points off the table are accessed.

(4) Values are transferred to the X (values surrounding P) and Y (values of the function F(P)) positions.

(5) The iteration is performed and the function F(P) is returned as needed data. Moments for certain thrust percentages and zero thrust axes positions are returned through the F(P) values.

APPENDIX C:

COMPUTER PROGRAM LISTINGS

C.1 PROGRAM FOR TOTALLY SYMMETRIC SECTIONS

```
      SUBROUTINE RSTRS
      SUBROUTINE XSTRS
      SUBROUTINE SDIAG
      SUBROUTINE SHIFT
      SUBROUTINE MOMNT
      SUBROUTINE THRST
      SUBROUTINE AITKN

      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TUD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY

      DRIVER 1
      DRIVER 3
      DRIVER 5
      DRIVER 7
      DRIVER 8
      DRIVER 9
      DRIVER 10
      DRIVER 11

      ASTEP=(TC(N)-PNA)/XN
      BSTEP=ASTEP
      WRITE(5,102)SF,PY,XMY,PHIY,PNA
      WRITE(5,105)
      READ(2,104)KF,KL,SF
33    CONTINUE
      DO 1000 LL=KF,KL
      SL=SF/(PHIY*LL)
      Z2=1./SL
      IF(XN-(FY(KK)/(E(KK)*Z2)+TC(KK)-PNA)/ASTEP)42,42,43
42    ASTEP=(FY(KK)/(E(KK)*Z2)+TC(KK)-PNA)/XN
43    NN=0
      PNC=PNA
12    NN=NN+1
      CALL SDIAG
      PNB=PNA
      PNA=PNC
      CALL MOMNT
      XM(NN)=XMM/XMY
      CALL THRST
      XT(NN)=THR/PY
      IF(NN-1)13,13,14
14    XT(NN)=THR/PY
      GO TO 15
13    XT(NN)=0.
```

```
15     XM(NN)=XMM/XMY
       IF(XT(NN)-.98)16,17,17
16     PNA=PNB
       PNA=PNA+ASTEP
       GO TO 12
17     IF(XT(NN-1.)18,18,19
18     NN=NN+1
19     XT(NN)=1.
       XM(NN)=0.
       DO 20 K=1,11,2
       K1=K/2+1
       P=(K-1)*.1
       CALL AITKN
20     CONTINUE
       Z2=Z2/PHIY
       WRITE(5,106)Z2,(Z(I),I=1,K1)
       ASTEP=BSTEP
1000   CONTINUE
       IF(SF-1.)30,30,31
31     READ(2,104)KF,KL,SF
       GO TO 33
30     CONTINUE
5000   CONTINUE

       DRIVER 12

       CALL EXIT
       END
// XEQ
```

C.2 PROGRAM FOR ASYMMETRIC SECTIONS

```
      SUBROUTINE RSTRS
      SUBROUTINE XSTRS
      SUBROUTINE SDIAG
      SUBROUTINE SHIFT
      SUBROUTINE MOMNT
      SUBROUTINE THRST
      SUBROUTINE AITKN

      DIMENSION ZAXIS(20),ZPHI(20)
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TUD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY

      DRIVER 1
      DRIVER 2
      DRIVER 3
      DRIVER 4
      DRIVER 5
      DRIVER 6
      DRIVER 7
      DRIVER 8
      DRIVER 9
      DRIVER 10
      DRIVER 11

      WRITE(5,102)SF,PY,XMY,PHIY,ECG,PCG
      STEP=ABS(ZCG-ECG)/20.
      IF(PCG-ZCG)23,38,24
23    D1=PCG
      D2=ZCG
      GO TO 29
38    PNA=ZCG
      GO TO 29
24    D1=ZCG
      D2=PCG
29    CONTINUE
      WRITE(5,105)
      IF(PCG-ECG)73,74,74
73    D3=PCG
      GO TO 75
74    D3=ECG
75    ASTEP=(TC(N)-D3)/XN
      BSTEP=ASTEP
      READ(2,104)KF,KL,SF
```

```
      K7=0
      K8=0
33    CONTINUE
      DO 1000 LL=KF,KL
      SL=SF/(PHIY*LL)
      Z2=1./SL
      IF(K8)83,83,45
83    IF(PHIY-Z2)21,22,22
21    PNA=D1-3*STEP
      N2=0
28    N2=N2+1
      CALL SDIAG
      CALL THRST
      XT(N2)=THR/PY
      XM(N2)=PNA
      PNA=PNA+STEP
      IF(PNA-D2-3*STEP)28,27,27
27    P=0.
      NN=N2
      K1=1
      CALL AITKN
      K7=K7+1
      ZAXIS(K7)=Z(1)
      ZPHI(K7)=SL
      PNA=Z(1)
      GO TO 45
22    PNA=ECG
45    IF(XN-(FY(KK)/(E(KK)*Z2)+TC(KK)-PNA)/ASTEP)42,42,43
42    ASTEP=(FY(KK)/(E(KK)*Z2)+TC(KK)-PNA)/XN
43    NN=0
      PNC=PNA
12    NN=NN+1
      CALL SDIAG
      PNB=PNA
      CALL THRST
      XT(NN)=THR/PY
      IF(PNA-PCG)76,76,77
77    PNA=PNA-(PNA-PCG)*XT(NN)
      GO TO 78
76    PNA=PNA+(PCG-PNA)*XT(NN)
78    CALL MOMNT
      XM(NN)=XMM/XMY
      IF(NN-1)13,13,14
14    XT(NN)=THR/PY
      GO TO 15
13    XT(NN)=0.
15    XM(NN)=XMM/XMY
      IF(XT(NN)-.98)16,17,17
```

```
16      PNA=PNB
        PNA=PNA+ASTEP
        IF(K8)90,90,12
17      IF(XT(NN)-1.)18,18,19
18      NN=NN+1
19      XT(NN)=1.
        XM(NN)=0.
90      CONTINUE
        DO 80 K=1,11,2
        K1=K/2+1
        P=(K-1)*.1
        CALL AITKN
        IF(K8)91,91,20
20      CONTINUE
91      Z2=Z2/PHIY
        WRITE(5,106)Z2,(Z(I),I=1,K1)
        ASTEP=BSTEP
1000    CONTINUE
        IF(K8)80,80,30
80      IF(SF-1.)81,81,31
31      READ(2,104)KF,KL,SF
        GO TO 33
81      IF(ECG-ZCG)87,87,84
87      CONTINUE
        DO 82 I=1,K7
        XT(I)=ZAXIS(I)
82      XM(I)=ZPHI(I)
        GO TO 86
84      CONTINUE
        DO 85 I=1,K7
        J=K7-I+1
        XT(J)=ZAXIS(I)
85      XM(J)=ZPHI(I)
86      P=PCG
        NN=K7
        K1=1
        CALL AITKN
        KF=1
        KL=1
        SF=Z(K1)*PHIY*KF
        K8=1
        PNA=PCG
        GO TO 33
30      CONTINUE
5000    CONTINUE

        DRIVER 12

        CALL EXIT
        END
```

C.3 SUBROUTINES

```
      SUBROUTINE SHIFT
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      M=K5
1     M=M+1
      IF(M-K)8,8,7
8     IF(HFY-H(M))2,7,1
2     K5=M
      K=K+1
      M=M+1
      DO 3 J1=M,K
      J2=K-J1+M
      J3=J2-1
      H(J2)=H(J3)
      S(J2)=S(J3)
3     W(J2)=W(J3)
      M=M-1
      IF(K6)5,5,6
5     H(M)=HFY
      W(M)=WD(I)
      C=HFY-PNA
      S(M)=SIGN(FY(I),C)
      GO TO 7
6     W(M)=W(M-1)
      H(M)=PNA
      Z(1)=PNA
      S(M)=0.
7     RETURN
      END
```

```
      SUBROUTINE SDIAG
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      K=1
      DO 200 I=1,N
      D=(TC(I)-BC(I))/XNN(I)
      L=0
      H(K)=BC(I)
      IF(NROPT(I))300,300,301
300   IF(NSOPT(I))203,203,301
203   STR=0.
      GO TO 220
301   NR(I)=1
404   IF(TC(I)-H(K))219,219,218
219   H(K)=TC(I)
      L=1
218   IF(NROPT(I))501,501,502
502   CALL RSTRS
      GO TO 503
501   STR=0.
503   IF(NSOPT(I))220,220,500
220   S(K)=E(I)*((H(K)-PNA)/SL+STR)
      SG=ABS(S(K))/S(K)
      IF(FY(I)-ABS(S(K)))201,202,202
201   S(K)=FY(I)*SG
      GO TO 202
500   STR=STR+(H(K)-PNA)/SL
      CALL XSTRS
202   W(K)=WD(I)
      K=K+1
      IF(L)204,204,200
204   IF(NR(I))211,211,212
212   H(K)=H(K-1)+D
      GO TO 404
211   H(K)=TC(I)
      L=1
      GO TO 218
200   CONTINUE
      K=K-1
      K5=0
      K6=0
      DO 205 I=1,N
      IF(NR(I))209,209,205
209   IF(PNA-BC(I))206,206,207
207   SG=-1.
      N1=0
      GO TO 4
```

```
206     IF(TC(I)-PNA)205,205,208
208     SG=1.
        N1=1
4       HFY=PNA+(FY(I)*SL*SG/E(I))
        IF(HFY-BC(I))1,1,2
2       IF(TC(I)-HFY)1,1,3
3       CALL SHIFT
1       IF(N1)206,206,205
205     CONTINUE
        K5=0
        K6=1
        HFY=PNA
        IF(HFY-TC(N))5,5,6
6       Z(1)=HFY
        RETURN
5       CALL SHIFT
        RETURN
        END


        SUBROUTINE XSTRS
        COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
       1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
       2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
       3E(5),TUD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
       4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
        S(K)=0.
        M=MATL(I)
        LB=NN2(M)
        A=STR/CSTR(M)
        J1=IABS(IFIX(A))
        IF(A)1,20,2
1       SG=-1.
        KS=-1
        GO TO 3
2       SG=1.
        KS=1
3       IF(LB-J1-1)19,19,4
4       J3=LB+J1*KS
        J2=JS+KS
        S(K)=VS(J3,M)+(VS(J2,M)-VS(J3,M))*(STR-VT(J3,M))/
       1(VT(J2,M)-VT(J3,M))
        GO TO 20
19      S(K)=FY(I)*SG
20      RETURN
        END
```

```
      SUBROUTINE MOMNT
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      XMM=0.
      DO 50 I=2,K
      J=I-1
      C=H(J)
      B=H(I)
      D=S(I)
      Q=S(J)
      IF(Z(1)-B)51,51,52
52    A=((B+C)/2.-PNA)*D+((2*C+B)/3.-PNA)*(Q-D)/2.
      GO TO 50
51    A=((B+C)/2.-PNA)*Q+((2*B+C)/3.-PNA)*(D-Q)/2.
50    XMM=XMM+(B-C)*(W(I)+W(J))*.5*A
      RETURN
      END
```

```
      SUBROUTINE THRST
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      THR=0.
      DO 30 I=2,K
      J=I-1
30    THR=THR-(S(I)+S(J))*(H(I)-H(J))*(W(I)+W(J))*.25
      RETURN
      END
```

```
      SUBROUTINE RSTRS
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,XMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN2(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      Q=H(K)-BC(I)
      STR=AR(I)*Q**2+BR(I)*Q+CR(I)
      RETURN
      END
```

```
      SUBROUTINE AITKN
      COMMON A,B,C,D,I,J,J1,J2,J3,K,K1,K5,K6,L,LB,M,N,NN,P,
     1PNA,Q,SL,SG,THR,KMM,STR,X(7),Y(7,7),Z(8),H(50),W(50),
     2S(50),XM(50),XT(50),BC(5),TC(5),TWD(5),WD(5),FY(5),
     3E(5),TWD(5),MATL(5),NN8(5),CSTR(5),AR(5),BR(5),CR(5),
     4XNN(5),NSOPT(5),NROPT(5),NR(5),VS(15,5),VT(15,5),HFY
      DO 207 I=1,NN
      IF(XT(I)-P)207,208,208
207   CONTINUE
      N1=0
800   N1=N1+1
      IF(XT(N1)-P)801,208,208
801   IF(P-XT(N1+1))802,208,800
802   LB=N1-2
      NUBAD=0
      IF(LB)803,803,804
803   NUBAD=2-N1
      LB=1
804   NUB=N1+3+NUBAD
      IF(NUB-NN)806,806,805
805   NUB=NN
806   KS=NUB-LB+1
      DO 807 I=1,KS
      J=LB+I-1
      X(I)=XT(J)
807   Y(I,1)=XM(J)
      DO 205 I=2,KS
      DO 205 J=I,KS
      L=I-1
205   Y(J,I)=(Y(L,L)*(X(J)-P)-Y(J,L)*(X(L)-P))/(X(J)-X(L))
      Z(K1)=Y(KS,KS)
      GO TO 204
208   Z(K1)=XM(I)
204   CONTINUE
      RETURN
      END
```

C.4 DRIVERS

DRIVER 1

```
      READ(2,100)NPROB
      DO 5000 LXXXX=1,NPROB
      WRITE(5,101)LXXXX,NPROB
      READ(2,104)N,NMTLS,MN
      DO 59 I=1,N
      MATL(I)=0
      NSOPT(I)=0
      NROPT(I)=0
59    NR(I)=0
      DO 1 I=1,N
      READ(2,103)BC(I),TC(I),WD(I),FY(I),E(I),AR(I),BR(I)
     1,CR(I)
1     WRITE(5,107)I,BC(I),TC(I),WD(I),FY(I),E(I),AR(I),BR(I)
     1,CR(I)
```

DRIVER 2

```
      EMIN=E(1)
      DO 2 I=2,N
      IF(EMIN-E(I))2,2,3
3     EMIN=E(I)
2     CONTINUE
      DO 4 I=1,N
4     TWD(I)=WD(I)*E(I)/EMIN
```

DRIVER 3

```
      PY=0.
      DO 5 I=1,N
5     PY=PY+(TC(I)-BC(I))*WD(I)*FY(I)
```

DRIVER 4

```
      THR=0.
      DO 70 I=1,N
      XMM=(TC(I)-BC(I))*WD(I)*FY(I)
      IF(THR+XMM-.5*PY)70,71,71
70    THR=THR+XMM
71    ZCG=BC(I)+(.5*PY-THR)/(FY(I)*WD(I))
```

DRIVER 5

```
      A=0.
      D=0.
      DO 6 I=1,N
      B=(TC(I)-BC(I))*TWD
      A=A+B
6     D=D+B*(TC(I)+BC(I))*.5
      ECG=D/A
```

DRIVER 6

```
      A=0.
      D=0.
      DO 7 I=1,N
      B=(TC(I)-BC(I))*WD(I)*FY(I)
      A=A+B
7     D=D+B*(TC(I)+BC(I))*.5
      PCG=D/A
```

DRIVER 7

```
      PHIY=FY(1)/(ECG*E(1))
      DO 8 I=1,N
      IF(FY(I)-ABS(ECG-BC(I))*PHIY*E(I))9,10,10
9     PHIY=FY(I)/ABS(E(I)*(ECG-BC(I)))
10    IF(FY(I)-ABS(ECG-TC(I))*PHIY*E(I))11,8,8
11    PHIY=FY(I)/ABS(E(I)*(ECG-TC(I)))
8     CONTINUE
```

DRIVER 8

```
      SL=1./PHIY
      PNA=ECG
      CALL SDIAG
      CALL MOMNT
      XMY=XMM
```

DRIVER 9

```
      D=0.
      DO 34 I=1,N
      IF(PCG-BC(I))36,36,35
35    IF(TC(I)-PCG)36,36,37
37    D=D+.5*WD(I)*FY(I)*((PCG-BC(I))**2+(TC(I)-PCG)**2)
      GO TO 34
36    D=D+FY(I)*WD(I)*(TC(I)-BC(I))*ABS(PCG-(TC(I)+BC(I))
     1*.5)
34    CONTINUE
```

DRIVER 10

```
      WRITE(5,112)
      DO 60 I=1,N
      READ(2,108)MATL(I),NROPT(I),NSOPT(I),NR(I),KNN(I)
60    WRITE(5,113)MATL(I),NROPT(I),NSOPT(I),KNN(I)
      IF(MATLS)56,56,57
57    WRITE(5,109)
      DO 58 I=1,NMTLS
      READ(2,100)NN2(I),CSTR(I)
      NK=2*NN2(I)-1
      K5=NK-1
      READ(2,103)(VS(J,I),J=1,NK),(VT(J,I),J=1,NK)
      WRITE(5,110)I,K5,CSTR(I),(VS(J,I),J=1,NK)
58    WRITE(5,111)(VT(J,I),J=1,NK)
```

DRIVER 11

```
56    SF=0.
      DO 40 I=1,N
      IF(SF-FY(I)/E(I))41,40,40
41    SF=FY(I)/E(I)
      KK=I
40    CONTINUE
```

DRIVER 12

```
100   FORMAT(I4,F10.0)
101   FORMAT(1H1,////,16X,"MOMENT-CURVATURE-THRUST DATA
     1 (MEMBER",I3," OF",I3,")",//,32X,"SECTION PARAMETERS"
     2,//,1X,"SECT.  BOTTOM     TOP  MAXIMUM  ELASTIC  SECT
     3.",10X,"RESIDUAL STRESS",/,3X,"NO.  HEIGHT  HEIGHT
     4  STRESS  MODULUS  WIDTH",13X,"CONSTANTS",/,9X,"(IN.
     5)    (IN.)    (KSI)     (KSI)   (IN.)",6X,"AR",9X,
     6"BR",9X,"CR")
102   FORMAT(1H0,15HSHAPE FACTOR   =,E14.6,/,1X15HTHRUST P(Y
     1)    =,E14.6,2X,"K",/,1X,15HMOMENT M(Y)     =,E14.6,2X,"
     2K-IN.",/,1X,15HANGLE PHIY    =,E14.6,2X,"RAD.",/,1X,1
     35HELASTIC C.G.   =,E14.6,"IN.",/,1X,15HPLASTIC C.G  =,
     4E14.6,2X,"IN.")
103   FORMAT(8F10.0)
104   FORMAT(2I4,F10.0)
105   FORMAT(1H0,12X,"M/M(Y) FOR  M/M(Y) FOR  M/M(Y) FOR  M
     1/M(Y) FOR  M/M(Y) FOR",/,1X,"PHI/PHIY   P/P(Y)=0.   P/
     2P(Y)=.2   P/P(Y)=.4   P/P(Y)=.6   P/P(Y)=.8   P/P(Y)=
     31.0")
106   FORMAT(1X,F9.3,4X,6(F8.4,4X),F10.4)
107   FORMAT(1X,I4,2X,F7.3,1X,F7.3,2X,F7.1,1X,F8.1,1X,F6.3,
     11X,3E11.3)
108   FORMAT(4I4,F10.0)
109   FORMAT(1H0,10X,"NONBILINEAR STRESS-STRAIN RELATIONSHIP
     1S")
110   FORMAT(1H0,"MATERIAL NO.",I3," ---",I3," LINEAR APPROX
     1IMATIONS ---DIVIDING STRAIN=",E11.3,"  IN.",/,1X,"STRE
     2SSES=",3X,9E11.3)
111   FORMAT(1X,"STRAINS =",3X,9E11.3)
112   FORMAT(1H0,"MATERIAL  RESIDUAL   NON-BILINEAR  SECTION
     1",/,3X,"NUMBER  STRESSES  STRESS-STRAIN  DIVIDER")
113   FORMAT(4X,I3,7X,I2,10X,I2,8X,F6.3)
```

C.5 CIRCULAR TUBE PROGRAM

```
      INTEGER UB,UBADD,SOPT,RSOPT
      DIMENSION W(220),S(220),H(220),Z(20)
      DIMENSION XT(100),XM(100),X(10),Y(10,10)
      COMMON VS(20),VT(20)
C *** R=OUTER RADIUS
C *** R1=INNER RADIUS
C *** B2=BOTTOM COORDINATE
C *** T=TOP COORDINATE
C *** F=YIELD STRESS
C *** E=ELASTIC MODULUS
C *** S1=STEP
C *** C1=RISE STEP
C *** XN=(DIMENSION OF A)-2
C *** H1=MEMBER CENTRAL HEIGHT
C *** P1=CRITICAL ANGLE
C *** P2=CRITICAL THRUST
C *** X1=CRITICAL MOMENT
C *** NDIM=DIMENSION NEEDED FOR W(),S(),H() IN CURRENT
C *** PROBLEM
C *** SOPT=NONBILINEAR STRESS-STRAIN OPTION
C *** RSOPT=RESIDUAL STRESS OPTION
C *** N2=SEGMENTS OF 1/2 STRESS-STRAIN DIAGRAM
C *** VS()=NONBILINEAR STRESSES
C *** VT()=NONBILINEAR STRAINS
C *** READ AND PRINT DATA, CALCULATE AND PRINT CONTROLERS
      READ(2,101)R,R1,B2,T,F,E,S1,XN
      READ(2,100)N2,SOPT,RSOPT,CSTR
C *** CSTR=DIVISIONAL STRAIN
      IF(SOPT)504,504,503
503   N3=2*N2-1
      READ(2,101)(VS(I),I=1,N3),(VT(J),J=1,N3)
504   NDIM=50
      H1=R
      P1=2*F/(E*T)
      X1=.785398163*F/R*(R**4-R1**4)
      P2=F*3.14159265*(R**2-R1**2)
      C1=T/(XN-2.)
      SFACT=(1.33333*(R**3-R1**3))/X1*F
      WRITE(5,102)R,R1,F,E,S1,SFACT,C1
      WRITE(5,103)P2,X1,P1
      IF(RSOPT)508,508,509
509   WRITE(5,106)
508   IF(SOPT)510,510,511
511   WRITE(5,107)
      DO 512 I=1,N3
512   WRITE(5,108)I,VS(I),VT(I)
510   CONTINUE
```

```
      WRITE(5,104)
C *** ZERO PERTINATE MATRIX ELEMENTS
      DO 223 I=1,NDIM
      W(I)=0.
      S(I)=0.
223   H(I)=0.
      KF=1
      KL=3
      SF=4.
224   CONTINUE
      DO 1000 LL=KF,KL
      S2=SF/(P1*LL)
C *** DETERMINE RANDOM THRUST-MOMENT VALUES
      N=0
      P=R
203   N=N+1
C *** BUILD STRESS DIAGRAM
      L1=0
      L2=0
      L5=0
      HT=0.
      K=0
      K4=1
      K=K+1
      L=0
      H(K)=B2
217   IF(L1)400,400,401
400   IF(R-R1-H(K))402,402,404
402   H(K)=R-R1
      L1=1
      GO TO 404
401   IF(L2)403,403,404
403   IF(R+R1-H(K))405,405,404
405   H(K)=R+R1
      L2=1
404   IF(T-H(K))219,219,218
219   H(K)=T
      L=1
505   IF(HSOPT)501,501,502
502   HT=H(K)
      CONTINUE
501   IF(SOPT)218,218,500
500   STR=(H(K)-P)/S2+HT
      CONTINUE
      S(K)=STR
      IF(L5)215,215,507
218   S(K)=E*(H(K)-P)/S2
      S3=ABS(S(K))/S(K)
      IF(F-ABS(S(K)))216,216,215
216   S(K)=F*S3
```

```
215   R2=(H1-H(K))**2
      IF(H1-R1-H(K))702,702,701
701   W(K)=2*SQRT(R**2-R2)
      GO TO 704
702   IF(H1+R1-H(K))701,701,703
703   W(K)=2*(SQRT(R**2-R2)-SQRT(R1**2-R1))
704   GO TO (705,717,718) ,K4
705   IF(L)214,214,200
214   K=K+1
      H(K)=H(K-1)+S1
      GO TO 217
200   CONTINUE
      K5=0
      K6=0
      IF(P-B2)302,302,301
301   S3=-1.
      K7=1
706   H2=P+(F*S2*S3/E)
      IF(H2-B2)707,707,708
708   IF(T-H2)707,707,709
709   GO TO 710
707   GO TO (302,300,719) ,K7
710   M=K5
711   M=M+1
      IF(M-K)720,720,719
720   IF(H2-H(M))712,713,711
712   K5=M
      K=K+1
      M=M+1
      DO 714 J1=M,K
      J2=K-J1+M
      J3=J2-1
      H(J2)=H(J3)
      S(J2)=S(J3)
714   W(J2)=W(J3)
      M=M-1
      J1=K
      K=M
      IF(K6)715,715,716
715   H(M)=H2
      K4=2
      GO TO 215
717   C=H2-P
      L5=1
      IF(SOPT)506,506,505
506   S(M)=SIGN(F,C)
507   K5=M
      L5=0
      GO TO 721
716   H(M)=P
```

```
      K4=3
      GO TO 215
718   IF(RSOPT)515,515,514
514   HT=P
      CONTINUE
      S(M)=E*HT
      GO TO 721
515   S(M)=0.
721   K=J1
713   GO TO (302,300,719) ,K7
302   IF(T-P)300,300,303
303   S3=1.
      K7=2
      GO TO 706
300   CONTINUE
      K6=1
      K5=0
      H2=P
      K7=3
      GO TO 710
C *** STRESS DIAGRAM COMPLETED,COMPUTE THRUST AND MOMENT
719   T1=0.
      X2=0.
      R4=P
      P=R
      DO 50 I=2,K
      J=I-1
      B=H(I)
      C=H(J)
      D=S(I)
      Q=S(J)
      W2=W(I)
      W1=W(J)
      T1=T1+(B-C)*(W1*(D+Q)/2.+(W2-W1)*(2.*D+Q)/6.)
      IF(B-P)51,51,52
52    X2=X2+(B-C)*(W1*Q*(.5*(C+B)-P)+.5*Q*(W2-W1)*((2.*B
     1+C)/3.-P)+.5*W1*(D-Q)*((2.*B+C)/3.-P)+(W2-W1)*(D-Q)
     2/3.*((3.*B+C)/4.-P))
      GO TO 50
51    X2=X2+(B-C)*(W2*D*(.5*(B+C)-P)+.5*D*(W1-W2)*((2.*C
     1+B)/3.-P)+.5*W2*(Q-D)*((2.*C+B)/3.-P)+(W1-W2)*(Q-D)
     2/3.*((3.*C+B)/4.-P))
50    CONTINUE
      T1=-T1
      P=R4
C *** ASSIGN RANDOM THRUST-MOMENT DATA TO MATRICES
      IF(N-1)220,220,221
221   XT(N)=T1/P2
      GO TO 222
220   XT(N)=0.
```

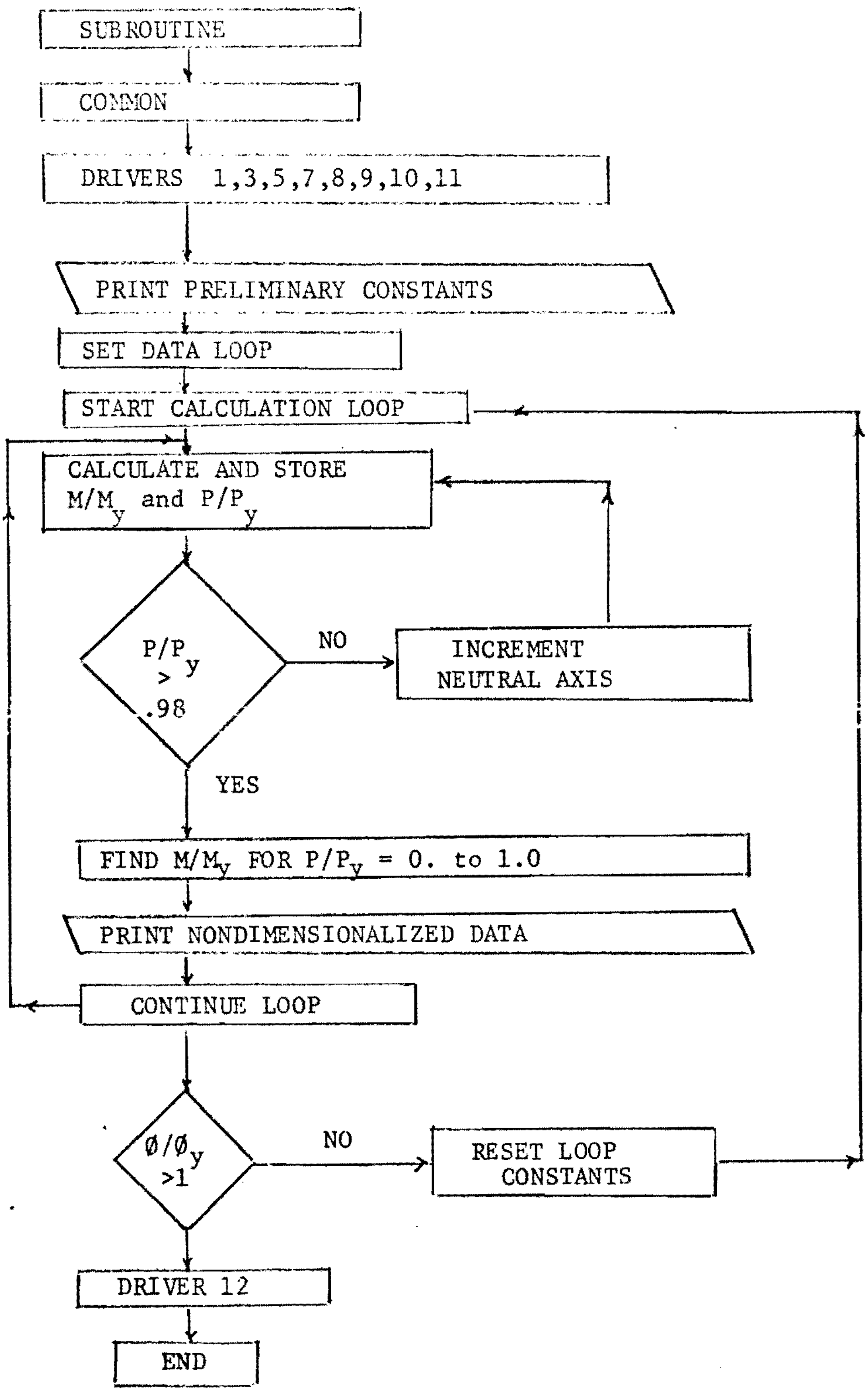
```
222     XM(N)=X2/X1
        IF(XT(N)-.98)201,202,202
201     P=P+C1
        GO TO 203
202     XT(N)=T1/P2
        XM(N)=X2/X1
        IF(XT(N)-1.)225,226,226
225     N=N+1
226     XT(N)=1.
        XM(N)=0.
C *** CALCULATE MOMENTS FOR GIEVN THRUSTS BY AITKEN'S
C *** PROCESS
        DO 204 K=1,11,2
        K1=K/2+1
        P=(K-1)*.1
C *** CHECK FOR EXACT VALUE FIT
        DO 207 I=1,N
        IF(XT(I)-P)207,208,207
207     CONTINUE
        N1=0
800     N1=N1+1
        IF(XT(N1)-P)801,208,208
801     IF(P-XT(N1+1))802,208,800
802     LB=N1-2
        UBADD=0
        IF(LB)803,803,804
803     UBADD=2-N1
        LB=1
804     UB=N1+3+UBADD
        IF(UB-N)806,806,805
805     UB=N
806     KS=UB-LB+1
        DO 807 I=1,KS
        J=LB+I-1
        X(I)=XT(J)
807     Y(I,1)=XM(J)
C *** NO EXACT FIT, PERFORM THE ITERATION
        DO 205 I=2,KS
        DO 205 J=I,KS
        L=I-1
205     Y(J,I)=(Y(L,L)*(X(J)-P)-Y(J,L)*(X(L)-P))/(X(J)-X(L))
C *** ASSIGN VALUES TO PRINT-OUT MATRICES,NONEXACT FIT
        Z(K1)=Y(KS,KS)
        GO TO 204
C *** EXACT FIT
208     Z(K1)=XM(I)
204     CONTINUE
        Z2=1./(P1*S2)
C *** PRINT MOMENT-CURVATURE-THRUST VALUES
        WRITE(5,105)Z2,(Z(I),I=1,K1)
```

```
1000  CONTINUE
      IF(SF-1.)1002,1002,1001
1001  KL=5
      SF=1.
      GO TO 224
1002  CONTINUE
100   FORMAT(3I4,F10.0)
101   FORMAT(8F10.0)
102   FORMAT('1',1X,'STRUCTURAL TUBE MOMENT-CURVATURE-THR
     1UST DATA',//,1X,22HOUTER RADIUS           =,E14.6,2X,
     2'IN.',/,1X,22HINNER RADIUS           =,E14.6,2X,'IN.'
     3,/,1X,22HYIELD(MAXIMUM) STRESS=,E14.6,2X,'KSI',/,1X
     4,22HMODULUS OF ELASTICITY=,E14.6,2X,'KSI',/,1X,22HS
     5ECTOR HEIGHT          =,E14.6,2X,'IN.',/,22HSHAPE FAC
     6TOR          =,E14.6,/,22HSTRAIN AXIS INCREMENT=,E14
     7.6,2X,'IN.')
103   FORMAT(1X,22HTHRUST P(Y)            =,E14.6,2X,'K',/,
     11X,22HMOMENT M(Y)          =,E14.6,2X,'K-IN.',/,1X,
     222HANGLE PHI(Y)         =,E14.6,2X,'RAD.')
104   FORMAT(1H0,'                 M/M(Y) FOR  M/M(Y) FOR  M/M
     1(Y) FOR  M/M(Y) FOR  M/M(Y) FOR  M/M(Y) FOR',/,'PHI
     2/PHI(Y)   P/P(Y)=0.    P/P(Y)=.2    P/P(Y)=.4    P/P(Y)
     3=.6    P/P(Y)=.8    P/P(Y)=1.0')
105   FORMAT(4X,F6.3,4X,6(F8.4,4X))
106   FORMAT(1H0,'*****RESIDUAL STRESS OPTION USED*****')
107   FORMAT(1H0,'*****NONBILINEAR STRESS OPTION USED****
     1*',/,1X,'STRESS-STRAIN CURVE DATA',/,1X,'POSITION',
     26X,'STRESS',11X,'STRAIN')
108   FORMAT(1X,I4,4X,E14.6,3X,E14.6)
      CALL EXIT
      END
```

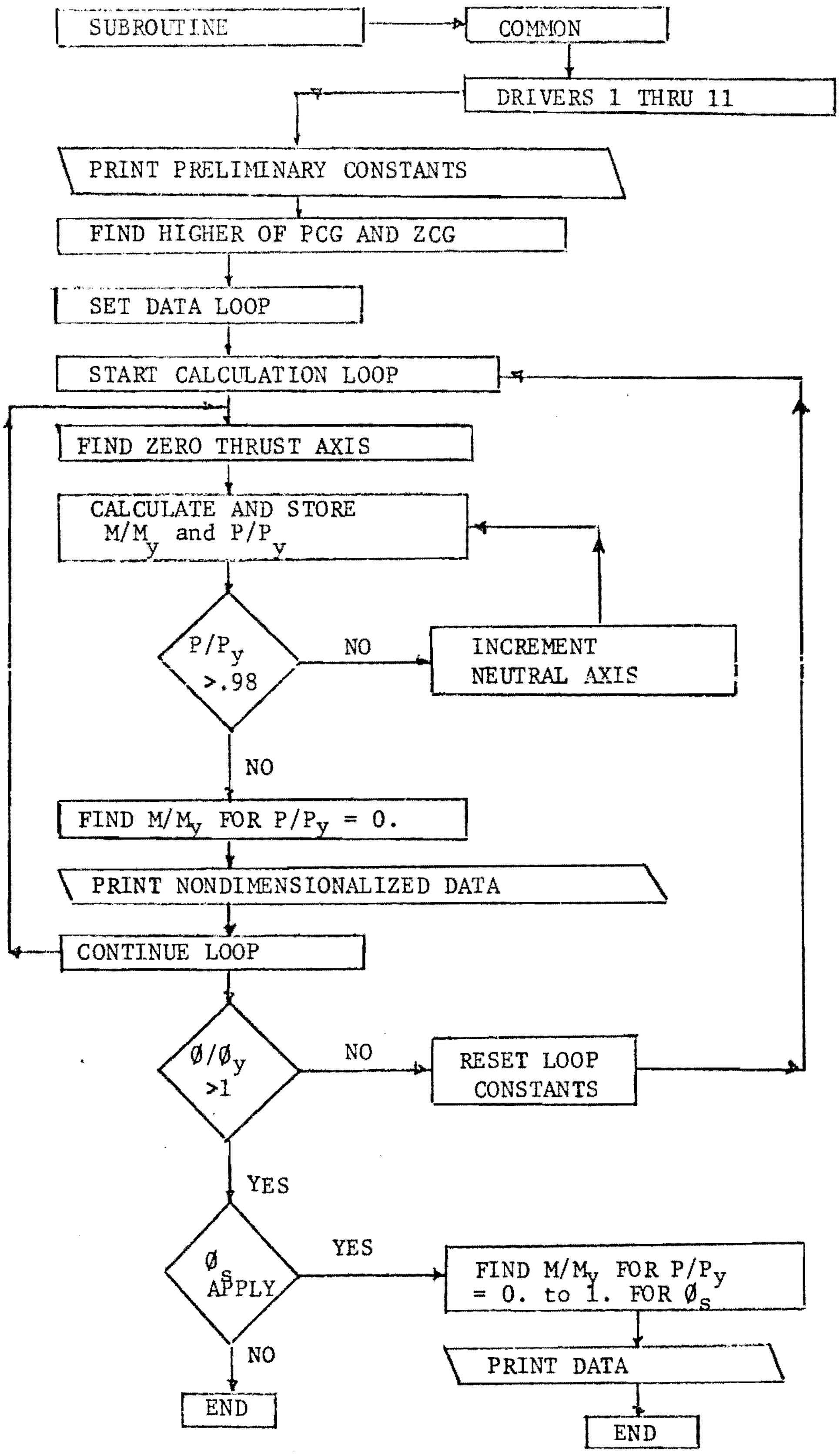
APPENDIX D:

FLOW DIAGRAMS

D.1 TOTALLY SYMMETRIC FLOW DIAGRAM

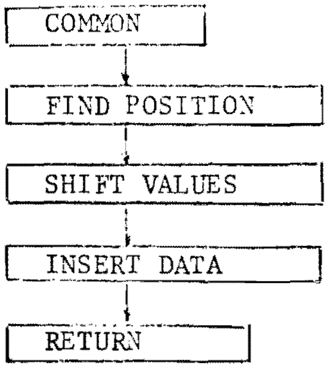


D.2 ASYMMETRIC FLOW DIAGRAM

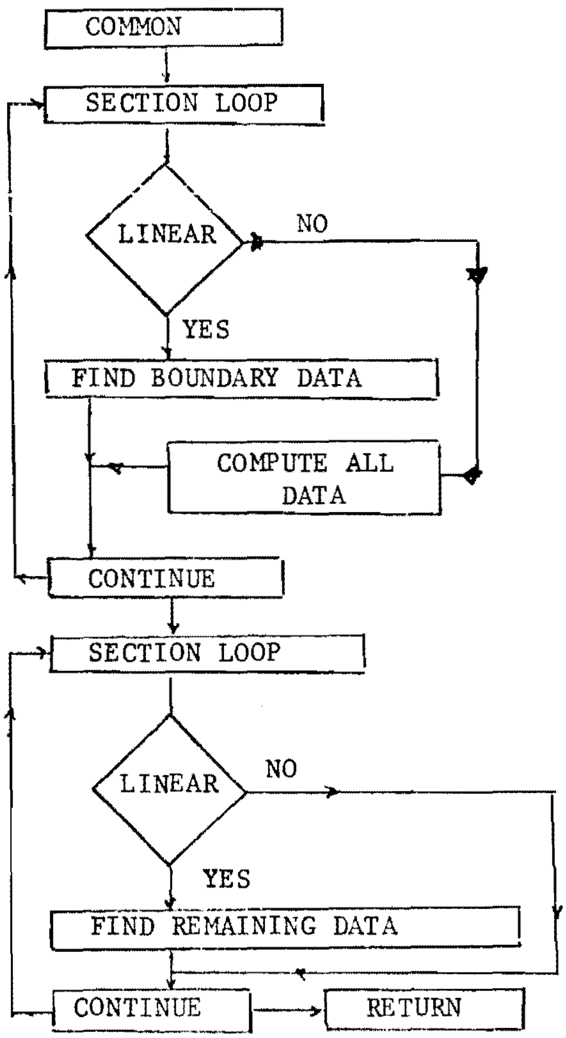


D.3 SUBROUTINES

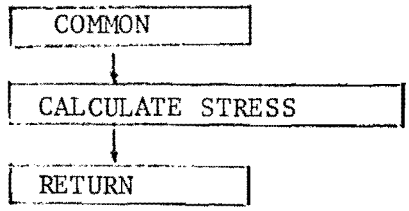
SHIFT



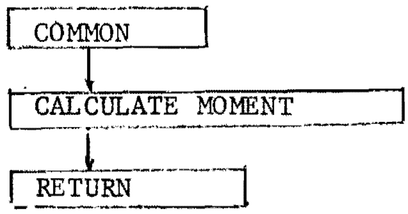
SDIAG



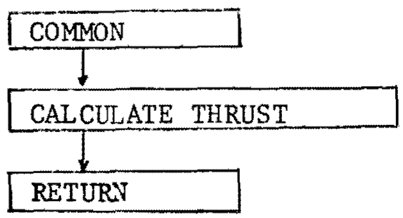
XSTRS



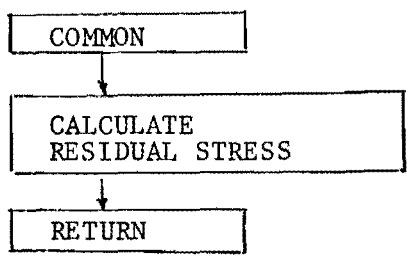
MOMNT



THRST



RSTRS



AITKN

COMMON

↓

EXACT DATA SEARCH

↓

FIND CLOSE DATA POINTS

↓

CHECK FOR OFF TABLE POINTS

↓

TRANSFER VALUES TO ITERATION TABLES

↓

PERFORM THE ITERATION

↓

RETURN NEEDED DATA

↓

RETURN

D.4 DRIVER FLOW DIAGRAMS

DRIVER 1

READ NO. OF PROBLEMS IN RUN

↓

START PROBLEM LOOP

↓

ZERO OPTION MATRICES

↓

READ AND PRINT SECTION DATA

↓

DRIVER 2

↓

DETERMINE EMIN

↓

CALCULATE TRANSFORMED WIDTHS

↓

DRIVER 3

↓

FIND THRUST P_y

↓

DRIVER 4

↓

DETERMINE ZCG

↓

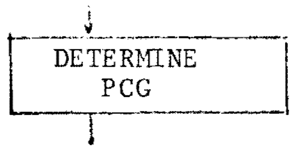
DRIVER 5

↓

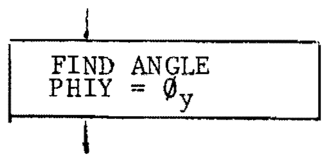
DETERMINE ECG

↓

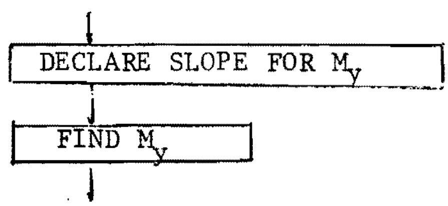
DRIVER 6



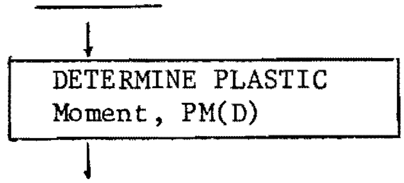
DRIVER 7



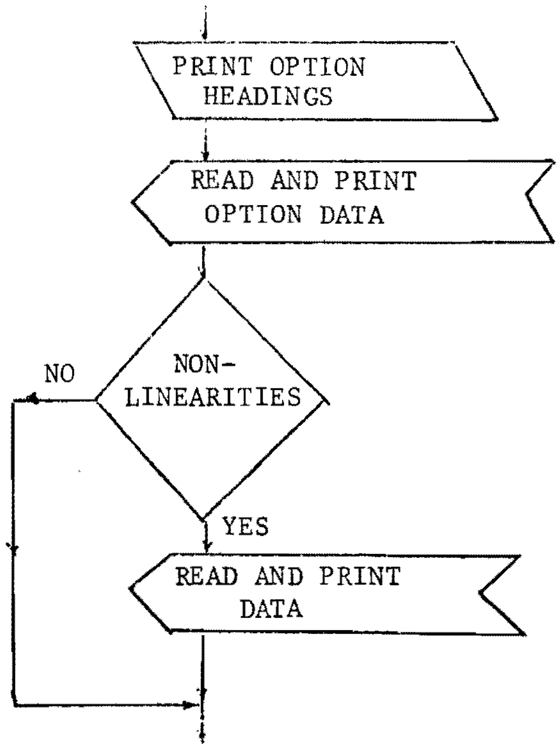
DRIVER 8



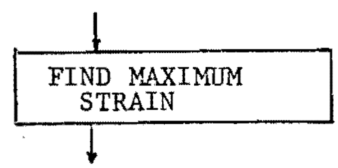
DRIVER 9



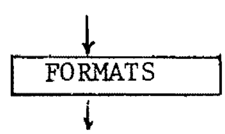
DRIVER 10



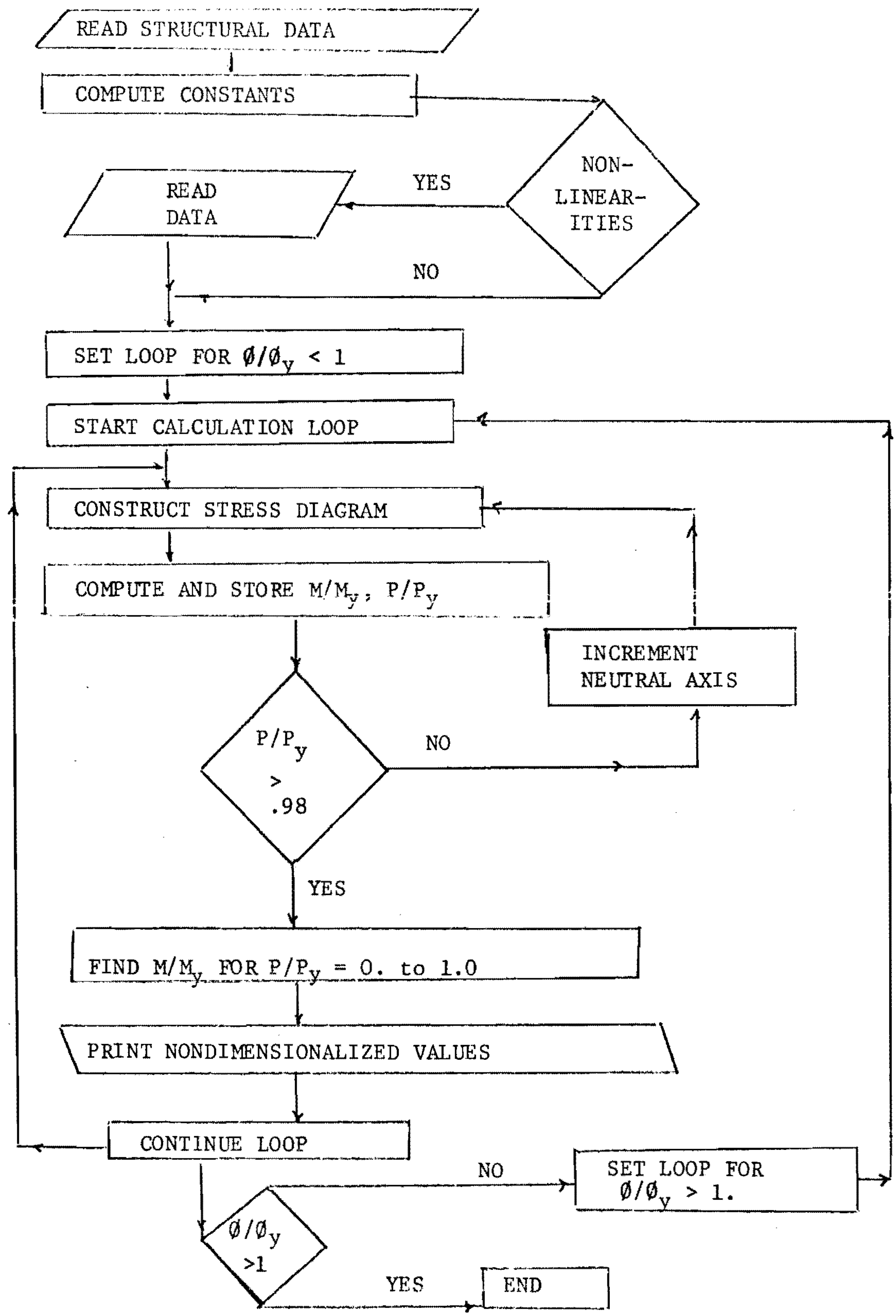
DRIVER 11



DRIVER 12



D.5 CIRCULAR TUBE FLOW DIAGRAM



VITA

Douglas Wrenn Fiala was born in Oregon City, Oregon, on November 18, 1946, the son of Jerry and Nellie Fiala. He graduated from West Linn High School in 1965 and received his Bachelor of Science in Applied Science with emphasis in Structural Engineering from Portland State University in June, 1970. He passed the Engineer-In-Training (EIT) Examination in April, 1970. Upon graduation he was appointed as a Graduate Teaching Assistant in the Department of Applied Science, Portland State University, and served in this capacity until June, 1972. Subsequently he accepted a position with Mackenzie Engineering Inc., Portland, Oregon, as a Structural Engineer. He is a member of The Society of the Sigma Xi.